CLASSICAL ELECTRODYNAMICS II

Homework Set 6

March 6, 2020

1. Derive a simple general expression for the vector potential in the radiation zone starting with

$$\mathbf{A}(\mathbf{r},t) = \frac{\mu_0}{4\pi} \int\int G^{(+)}(\mathbf{r},t;\mathbf{r}',t')\mathbf{J}(\mathbf{r}',t')\; d^3r'\; dt' \;,$$

where $G^{(+)}(\mathbf{r},t;\mathbf{r}',t')$ is the retarded (causal) Green function. That is, do *not* assume $\mathbf{J}(\mathbf{r},t) = \mathbf{J}(\mathbf{r})\; \mathrm{e}^{-\mathrm{i}\omega t}$.

2. Now consider a rotating electric dipole consisting of two equal and opposite charges q and $-q$ attached to the ends of a rod of length s. The rod rotates counterclockwise in the x-y plane with angular speed $\omega = ck$. The electric dipole moment of the system at $t = 0$ has the value $\mathbf{p}_0 = qs\hat{x}$. Use your result from part (a) to calculate $\mathbf{A}(\mathbf{r},t)$ in the radiation zone. (*Hint*: Recall that $ks \ll 1$.) Show that your result can be put in the complex form:

$$\mathbf{A}(\mathbf{r},t) = \frac{\mu_0}{4\pi r}\, p_0\omega \left(\hat{\phi} - \mathrm{i}\hat{\rho}\right) \mathrm{e}^{\mathrm{i}(\phi - \omega t + kr)} \;,$$

where $\hat{\rho} = \hat{x}\cos\phi + \hat{y}\sin\phi$ and $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$.

3. Use the complex expression in part (b) to calculate $\mathbf{B}(\mathbf{r},t)$ in the radiation zone.

4. Use your result of part (c) to calculate $\mathbf{E}(\mathbf{r},t)$ in the radiation zone.

5. Use your results of parts (c) and (d) to determine equations for the real, physical fields $\mathbf{E}$ and $\mathbf{B}$. Then calculate the instantaneous Poynting vector. Is the instantaneous Poynting vector azimuthally symmetric (independent of ϕ)? If not, are there any observation points where it is independent of ϕ? Finally, calculate the time-averaged Poynting vector and comment on whether it depends on ϕ or not.

6. Lastly calculate the time-averaged power radiated per unit solid angle and make a sketch of the radiation pattern.