CLASSICAL ELECTRODYNAMICS II

Homework Set 8

April 20, 2018

1. A coordinate system S' moves with a velocity $\mathbf{v}$ relative to another sytem S. In S' a particle has a velocity $\mathbf{u}'$ and an acceleration $\mathbf{a}'$. In Homework Set 7, you were asked to show that in system S the component of acceleration parallel to $\mathbf{v}$ is

$$\mathbf{a}_{\parallel} = \frac{\left(1 - \dfrac{v^2}{c^2}\right)^{3/2}}{\left(1 + \dfrac{\mathbf{v}\cdot\mathbf{u}'}{c^2}\right)^3}\,\mathbf{a}'_{\parallel} \ .$$

Now assume that a rocket ship leaves the Earth in the year 2018. The rocket ship is so constructed that it has an acceleration g in its own rest frame (this makes the occupants feel at home). It accelerates in a straight-line path for 2.5 years (by its own clocks), decelerates at the same rate for 2.5 more years, turns around, accelerates for 2.5 more years, decelerates for 2.5 years, and lands on Earth.

(a) What year is it on Earth?

(b) How far away from the Earth did the rocket ship travel? Give your answer in light years.

2. A particle of mass m and charge q moves in a laboratory in static, uniform electric and magnetic fields. Both $\mathbf{E}$ and $\mathbf{B}$ are parallel to the z axis. Show that with appropriate constants of integration, the parametric solution can be written as

$$\begin{aligned}
x &= \left(\frac{p_\perp c}{qB}\right)\sin\phi \ , \\
y &= \left(\frac{p_\perp c}{qB}\right)\cos\phi \ , \\
z &= \frac{m_\perp c^2}{qE}\cosh\left(\frac{E}{B}\phi\right) \ , \\
t &= \frac{m_\perp c}{qE}\sinh\left(\frac{E}{B}\phi\right) \ .
\end{aligned}$$

Here $m_\perp$ and $p_\perp$ are constants called the transverse mass and transverse momentum, respectively. The parameter ϕ is simply related to the proper time τ; find the relation. Also determine how p_x, p_y, p_z, and the total relativistic energy $\mathcal{E}$ depend on ϕ.