CLASSICAL ELECTRODYNAMICS II

Homework Set 4

February 23, 2018

1. Consider a monochromatic plane wave propagating along the z axis in an isotropic nonpermeable ($\mu = \mu_0$) dielectric. If the dielectric is a gyrotropic material that has been placed in a static external magnetic field, then the electric displacement vector can be written as

$$\mathbf{D} = \epsilon \mathbf{E} + \mathrm{i} \mathbf{E} \times \mathbf{g} \ ,$$

   where the permittivity ϵ is a positive real number and $\mathbf{g}$ is a constant real vector (called the gyration vector), which is proportional to the applied magnetic field. If the applied magnetic field is along the direction of propagation, then $\mathbf{g} = g\hat{z}$. The index of refraction for the medium can be written as $n = ck/\omega$, where ω is the frequency of the propagating wave and k is its wave number.

   (a) Show that this medium is birefringent (double refracting) with two indices of refraction:

$$n = \left(\frac{\epsilon \pm g}{\epsilon_0} \right)^{1/2} \ .$$

   (b) Determine the electric field for each value of n and describe the corresponding polarization.

2. In class, I derived the dispersion relation:

$$\mathrm{Re}\, \frac{\epsilon(\omega)}{\epsilon_0} = 1 + \frac{2}{\pi}\, P \int_0^\infty \frac{\omega' \ \mathrm{Im}\ \epsilon(\omega')/\epsilon_0}{\omega'^2 - \omega^2}\, d\omega' \ .$$

   Use this dispersion relation to calculate Re $\epsilon(\omega)/\epsilon_0$, given the imaginary part of $\epsilon(\omega)$ for positive ω as

$$\mathrm{Im}\, \frac{\epsilon(\omega)}{\epsilon_0} = \frac{\lambda \gamma \omega}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2} \ ,$$

   where λ and γ are constants.