CLASSICAL ELECTRODYNAMICS II

Homework Set 2

February 9, 2018

1. In class, we considered how Maxwell's equations in a medium would be modified if magnetic monopoles existed:

$$
\begin{aligned}
\nabla \cdot \mathbf{D} &= \rho_e, \\
\nabla \cdot \mathbf{B} &= \rho_m, \\
\nabla \times \mathbf{H} &= \mathbf{J}_e + \frac{\partial \mathbf{D}}{\partial t}, \\
\nabla \times \mathbf{E} &= -\mathbf{J}_m - \frac{\partial \mathbf{B}}{\partial t}.
\end{aligned}
$$

   Show explicitly that these equations are invariant under a duality transformation, as defined in class.

2. Consider electromagnetic plane waves propagating in a medium in which $\mathbf{D} = \epsilon \mathbf{E}$ and $\mathbf{B} = \mu \mathbf{H}$, where the fields are given by the complex representation:

$$
\mathbf{E} = \mathbf{E}_0 \, \mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r} - \omega t)} \; ,
$$
$$
\mathbf{B} = \mathbf{B}_0 \, \mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r} - \omega t)} \; ,
$$

   with $\mathbf{E}_0$ and $\mathbf{B}_0$ complex. Show explicitly that the time-averaged Poynting vector is given by

$$
\mathbf{S} = \frac{1}{2} \mathrm{Re}(\mathbf{E} \times \mathbf{H}^*) \; .
$$

3. Consider electromagnetic waves in source-free space where $\epsilon = \epsilon_0$ and $\mu = \mu_0$. Given the explicitly real field $\mathbf{E}$ for each part below, calculate the corresponding magnetic induction $\mathbf{B}$, the Poynting vector, $\mathbf{S} = \mathbf{E}\times\mathbf{B}/\mu_0$, and the time-averaged Poynting vector. Interpret each case using, as appropriate, the following descriptors: traveling wave; standing wave; plane wave; spherical wave; linearly polarized wave; circularly polarized wave; elliptically polarized wave.

   (a) $\mathbf{E} = \mathbf{E}_0 \sin(\mathbf{k}\cdot\mathbf{r} - \omega t)$

   (b) $\mathbf{E} = \mathbf{E}_0 \sin(kr)\sin(\omega t)$