CLASSICAL ELECTRODYNAMICS II

Homework Set 8

November 4, 2016

1. Consider the electric quadrupole potential,

$$\mathbf{A}_{\mathrm{S}}(\mathbf{r}) = -\frac{\mu_0 k^2 c}{24\pi}\left(\frac{\mathrm{e}^{ikr}}{r}\right)\left[\mathbf{Q}(\hat{r}) + \hat{r}\int r'^2\ \rho(r')\ d^3r'\right] .$$

   Show explicitly that the corresponding magnetic field in the radiation zone is

$$\mathbf{H} = \mathrm{i}k\hat{r} \times \mathbf{A}_S/\mu_0 .$$

2. Consider two inertial frames S and S', which recede from each other with constant velocity $\mathbf{u} = \boldsymbol{\beta}c$. A 4-vector is defined as a set of four real numbers that transforms the same as the set of coordinates $(ct, x, y, z) = (ct, \mathbf{r})$. The Lorentz transformation equations for an arbitrary 4-vector $A = (A_0, \mathbf{A})$ can be written as

$$\begin{aligned} A_0' &= \gamma(A_0 - \boldsymbol{\beta}\cdot\mathbf{A}) , \\ \mathbf{A}_{\parallel}' &= \gamma(\mathbf{A}_{\parallel} - \boldsymbol{\beta}A_0) , \\ \mathbf{A}_{\perp}' &= \mathbf{A}_{\perp} , \end{aligned}$$

   where $\mathbf{A} = \mathbf{A}_{\parallel} + \mathbf{A}_{\perp}$, with $\mathbf{A}_{\parallel}$ and $\mathbf{A}_{\perp}$ the components of $\mathbf{A}$ that are parallel and perpendicular to $\boldsymbol{\beta}$, respectively. Use the information given to show that the 3-vector $\mathbf{A}$ transforms under a Lorentz transformation according to

$$\mathbf{A}' = \mathbf{A} + \frac{\gamma - 1}{\beta^2}\,(\mathbf{A}\cdot\boldsymbol{\beta})\boldsymbol{\beta} - \gamma\boldsymbol{\beta}A_0 .$$

3. Use the general transformation equations for A_0 and $\mathbf{A}$ given in problem 2 to show explicitly that

$$A_0'^2 - \mathbf{A}'^2 = A_0^2 - \mathbf{A}^2 .$$

   That is, the quantity $A_0^2 - \mathbf{A}^2$ is a *Lorentz scalar* - it has the same value in *all* inertial frames.