CLASSICAL ELECTRODYNAMICS II

Homework Set 6

October 21, 2016

1. A plane wave of frequency ω is incident normally (along $\hat{z}$) from vacuum on a semi-infinite slab of material with a *complex* index of refraction $n(\omega)$.

   (a) Show that the ratio of reflected power to incident power is

   $$R = \left| \frac{1 - n(\omega)}{1 + n(\omega)} \right|^2 .$$

   (b) Show that the ratio of transmitted power to incident power is

   $$T = \frac{4 \, \mathrm{Re} \, n(\omega)}{|1 + n(\omega)|^2} .$$

   Hint: R and T are the reflection and transmission coefficients, respectively, defined here as

   $$R = \left| \frac{(\mathbf{S}'' \cdot \hat{z})}{(\mathbf{S} \cdot \hat{z})} \right| \qquad T = \left| \frac{(\mathbf{S}' \cdot \hat{z})}{(\mathbf{S} \cdot \hat{z})} \right| ,$$

   where $\mathbf{S}$, $\mathbf{S}'$, and $\mathbf{S}''$ are the time-averaged Poynting vectors for the incident, reflected, and refracted waves, respectively.

2. Use the dispersion relation,

   $$\mathrm{Re} \, \frac{\epsilon(\omega)}{\epsilon_0} = 1 + \frac{2}{\pi} \, P \int_0^\infty \frac{\omega' \, \mathrm{Im} \, \epsilon(\omega')/\epsilon_0}{\omega'^2 - \omega^2} \, d\omega' .$$

   to calculate $\mathrm{Re} \, \epsilon(\omega)$, given the imaginary part of $\epsilon(\omega)$ for positive ω as

   $$\mathrm{Im} \, \frac{\epsilon(\omega)}{\epsilon_0} = \lambda \left[\theta(\omega - \omega_1) - \theta(\omega - \omega_2) \right] ,$$

   where $\omega_2 > \omega_1 > 0$. Here $\theta(\tau)$ is the step function defined such that $\theta(\tau) = 0$ for $\tau < 0$ and $\theta(\tau) = 1$ for $\tau > 0$. Sketch the behavior of $\mathrm{Im} \, \epsilon(\omega)$ and the result for $\mathrm{Re} \, \epsilon(\omega)$ as functions of ω.