CLASSICAL ELECTRODYNAMICS II

Homework Set 4

September 30, 2016

1. Consider a material in which the half-space with $z > 0$ is filled with a medium with permeability μ' and dielectric constant ϵ', and the half-space with $z < 0$ is filled with a medium with permeability μ and dielectric constant ϵ. A electromagnetic wave with electric field,

$$\mathbf{E} = \mathbf{E}_0 \, \mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r}-\omega\mathrm{t})} \ ,$$

is incident on the interface at $z = 0$ between the two dielectrics. Consider the case where $\mathbf{E}$ is parallel to the plane of incidence. Then, starting with the boundary conditions on the fields as derived in class, derive expressions for the ratios E_0'/E_0 and E_0''/E_0, where E_0' is the amplitude of the electric field of the transmitted wave and E_0'' is the amplitude of electric field of the reflected wave.

2. The most general homogeneous plane wave propagating in the direction $\mathbf{k}$ may be represented as a superposition of two circularly polarized waves:

$$\mathbf{E}(\mathbf{r}, t) = (\hat{\epsilon}_+ E_+ + \hat{\epsilon}_- E_-) \, \mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r}-\omega t)} \ ,$$

where E_+ and E_- are complex amplitudes. Show that if $E_-/E_+ = r \, e^{\mathrm{i}\alpha}$, where r and α are real then the $\mathbf{E}$ vector traces out an ellipse with ratio of semimajor axis to semiminor axis, $(r+1)/(r-1)$, and the axes of the ellipse are rotated by an angle $\alpha/2$ relative to $\hat{\epsilon}_1$ and $\hat{\epsilon}_2$. For convenience, assume that $r > 1$.