CLASSICAL ELECTRODYNAMICS II

Homework Set 2

September 16, 2016

1. A transverse wave is incident normally in vacuum on a perfectly absorbing flat screen. Use the law of conservation of linear momentum to show that the pressure (called radiation pressure) exerted on the screen is equal to the field energy per unit volume in the wave.

2. Consider a finite volume V that contains both charged particles and electromagnetic fields. Assume that no particles enter or leave V. Show that the Minkowski expressions for the differential and integral forms of the conservation of angular momentum law are

$$\frac{\partial}{\partial t}(\mathcal{L}_{\text{mech}} + \mathcal{L}_{\text{field}}) + \nabla \cdot \overleftrightarrow{\mathbf{M}} = 0$$

and

$$\frac{d}{dt}\int_V (\mathcal{L}_{\text{mech}} + \mathcal{L}_{\text{field}}) d\tau + \oint_S \overleftrightarrow{\mathbf{M}} \cdot \vec{da} = 0$$

where the field angular-momentum density is

$$\mathcal{L}_{\text{field}} = \mathbf{r} \times \mathbf{g} \ ,$$

with $\mathbf{g} = \mathbf{D} \times \mathbf{B}$ the Minkowski expression for the electromagnetic linear momentum density, and the flux of angular momentum is described by the tensor

$$\overleftrightarrow{\mathbf{M}} = \overleftrightarrow{\mathbf{T}} \times \mathbf{r} \ ,$$

where $\overleftrightarrow{\mathbf{T}}$ is the Maxwell stess tensor.