CLASSICAL ELECTRODYNAMICS II
Homework Set 8
November 7, 2014

1. Consider the electric quadrupole potential,

$$\mathbf{A}_{\mathrm{S}}(\mathbf{r}) = -\frac{\mu_0 k^2 c}{24\pi} \left(\frac{\mathrm{e}^{\mathrm{i}kr}}{r} \right) \left[\mathbf{Q}(\hat{r}) + \hat{r} \int r'^2\, \rho(r')\, d^3r' \right] \ .$$

   Show explicitly that the corresponding magnetic field in the radiation zone is

   $$\mathbf{H} = \mathrm{i}k\hat{r} \times \mathbf{A}_S / \mu_0 \ .$$

2. Consider two inertial frames S and S', which recede from each other with constant velocity $\mathbf{u} = \boldsymbol{\beta}c$. A 4-vector is defined as a set of four real numbers that transforms the same as the set of coordinates $(ct, x, y, z) = (ct, \mathbf{r})$. The Lorentz transformation equations for an arbitrary 4-vector $A = (A_0, \mathbf{A})$ can be written as

   $$\begin{aligned}
   A_0' &= \gamma(A_0 - \boldsymbol{\beta} \cdot \mathbf{A}) \ , \\
   \mathbf{A}_\parallel' &= \gamma(\mathbf{A}_\parallel - \boldsymbol{\beta}A_0) \ , \\
   \mathbf{A}_\perp' &= \mathbf{A}_\perp \ ,
   \end{aligned}$$

   where $\mathbf{A} = \mathbf{A}_\parallel + \mathbf{A}_\perp$, with $\mathbf{A}_\parallel$ and $\mathbf{A}_\perp$ the components of $\mathbf{A}$ that are parallel and perpendicular to $\boldsymbol{\beta}$, respectively. Use the information given to show that the 3-vector $\mathbf{A}$ transforms under a Lorentz transformation according to

   $$\mathbf{A}' = \mathbf{A} + \frac{\gamma - 1}{\beta^2}\, (\mathbf{A} \cdot \boldsymbol{\beta})\boldsymbol{\beta} - \gamma\boldsymbol{\beta}A_0 \ .$$

3. Use the general transformation equations for A_0 and $\mathbf{A}$ given in problem 2 to show explicitly that

   $$A_0'^2 - \mathbf{A}'^2 = A_0^2 - \mathbf{A}^2 \ .$$

   That is, the quantity $A_0^2 - \mathbf{A}^2$ is a *Lorentz scalar* - it has the same value in *all* inertial frames.