CLASSICAL ELECTRODYNAMICS II

Homework Set 3

September 19, 2014

1. Consider a monochromatic plane wave propagating along the z axis in an isotropic nonpermeable ($\mu = \mu_0$) dielectric. If the dielectric is a gyrotropic material that has been placed in a static external magnetic field, then the electric displacement vector can be written as

$$\mathbf{D} = \epsilon \mathbf{E} + \mathrm{i}\mathbf{E} \times \mathbf{g} \ ,$$

where the permittivity ϵ is a positive real number and $\mathbf{g}$ is a constant real vector (called the gyration vector), which is proportional to the applied magnetic field. If the applied magnetic field is along the direction of propagation, then $\mathbf{g} = g\hat{z}$. The index of refraction for the medium can be written as $n = ck/\omega$, where ω is the frequency of the propagating wave and k is its wave number. Show that this material is birefringent (double refracting) with two indices of refraction and determine their values.

2. Consider electromagnetic waves in source-free space where $\epsilon = \epsilon_0$ and $\mu = \mu_0$. Given the explicitly real field $\mathbf{E}$ for each part below, calculate the corresponding magnetic induction $\mathbf{B}$, the Poynting vector, $\mathbf{S} = \mathbf{E}\times\mathbf{B}/\mu_0$, and the time-averaged Poynting vector. Interpret each case using, as appropriate, the following descriptors: traveling wave; standing wave; plane wave; spherical wave; linearly polarized wave; circularly polarized wave; elliptically polarized wave. The time-averaged Poynting vector is zero for a standing wave.

   (a) $\mathbf{E} = \mathbf{E}_0 \sin(\mathbf{k}\cdot\mathbf{r} - \omega t)$

   (b) $\mathbf{E} = \mathbf{E}_0 \sin(kr - \omega t)$

   (c) $\mathbf{E} = \mathbf{E}_0 \sin(\mathbf{k}\cdot\mathbf{r}) \sin(\omega t)$

   (d) $\mathbf{E} = E_0\ \hat{x}\cos(kz - \omega t) + E_0\ \hat{y}\sin(kz - \omega t)$

   (e) $\mathbf{E} = 3E_0\ \hat{x}\cos(kz - \omega t) + 2E_0\ \hat{y}\sin(kz - \omega t)$

   (f) $\mathbf{E} = E_0\ [\hat{x}\cos(\omega t) + \hat{y}\sin(\omega t)] \sin(kz)$