CLASSICAL ELECTRODYNAMICS II

Homework Set 1

September 5, 2014

1. (a) Consider a disk of radius R that lies in the x-y plane and carries a uniform charge per unit area σ. Let Q be the total charge on the disk. Determine the electrostatic potential anywhere on the z axis.

   (b) Show that the potential $\Phi(r,\theta)$ for $r > R$ is given by

   $$\Phi(r,\theta) = \frac{Q}{4\pi\epsilon_0}\frac{1}{r}\sum_{\ell=0}^{\infty}\left(\frac{R}{r}\right)^{\ell}\frac{2}{\ell+2}P_\ell(0)P_\ell(\cos\theta)\ ,$$

   where $P_\ell(x)$ is a Legendre polynomial. Note that $P_\ell(1) = 1$.

   (c) Also determine the potential $\Phi(r,\theta)$ for $r < R$.
   Hint: For $0 < x < 1$,

   $$\frac{1}{\sqrt{1+x^2}} = \sum_{\ell=0}^{\infty} P_\ell(0)x^\ell\ .$$

2. The magnetic induction at point P produced by an arbitrary current loop carrying current I can be written as

   $$\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I}{4\pi}\oint_C \frac{d\mathbf{r}'\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\ ,$$

   where the current element $I d\mathbf{r}'$ is located at position $\mathbf{r}'$ and point P is located at position $\mathbf{r}$. Now consider a circular current loop of radius a which lies in the $z = 0$ plane with its center at the origin. Derive an expression for the magnetic induction, $\mathbf{B}(z)$, on the z-axis of the loop. That is, calculate $\mathbf{B}(\mathbf{r})$ at $\mathbf{r} = z\hat{z}$. Do this by first calculating the contribution from the top half of the loop (region $0 < \theta < \pi$) and then the contribution from the bottom half (region $\pi < \theta < 2\pi$). Finally add the two contributions together to get the total, which (by symmetry) should have only a z-component. Note: If all I wanted was the final result, it would be much simpler just to calculate

   $$\mathbf{B}(z) = \frac{\mu_0 I}{4\pi}\hat{z}\oint_C \hat{z}\cdot\frac{d\mathbf{r}'\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\ .$$