CLASSICAL ELECTRODYNAMICS II

Homework Set 5

October 23, 2009

1. A common textbook example of a radiating system is a configuration of charges fixed relative to each other but in rotation. The charge density is obviously a function of time, but is not of the form $\rho(\mathbf{r},t) = \rho(\mathbf{r})\ \mathrm{e}^{-\mathrm{i}\omega t}$.

   (a) Consider a charge density $\rho(\mathbf{r},t)$ that is periodic in time with period $T = 2\pi/\omega_0$. By making a Fourier *series* expansion, show that it can be written as

   $$\rho(\mathbf{r},t) = \rho_0(\mathbf{r}) + \sum_{n=1}^{\infty} \mathrm{Re}\ [2\rho_n(\mathbf{r})\mathrm{e}^{-\mathrm{i}n\omega_0 t}]\ ,$$

   where

   $$\rho_n(\mathbf{r}) = \frac{1}{T}\int_0^T \rho(\mathbf{r},t)\mathrm{e}^{\mathrm{i}n\omega_0 t}\ dt\ .$$

   (b) Express the charge density $\rho_n(\mathbf{r})$ in cylindrical coordinates for a single charge q rotating about the origin in the x-y plane in a circle of radius R at constant angular speed ω_0.

   (c) Calculate the electric dipole moment $\mathbf{p}$ for the rotating charge in part (b).

   (d) Calculate the nonzero components of the electric quadrupole tensor

   $$Q_{ij} = \int (3x_i'x_j' - r'^2\delta_{ij})\ \rho(\mathbf{r}',t)\ d^3r'$$

   for the rotating charge in part (b).

   (e) Calculate the magnetic dipole moment

   $$\mathbf{m} = \frac{1}{2}\int [\mathbf{r}' \times \mathbf{J}(\mathbf{r}',t)]\ d^3r'$$

   for the rotating charge in part (b).

   (f) For which of the multipoles in parts (c), (d), and (e) will the rotating charge produce radiation? At what frequencies?