CLASSICAL ELECTRODYNAMICS II

Homework Set 3

October 2, 2009

1. Consider a material in which the half-space with $z > 0$ is filled with a medium with permeability μ' and dielectric constant ϵ', and the half-space with $z < 0$ is filled with a medium with permeability μ and dielectric constant ϵ. A electromagnetic wave with electric field,

$$\mathbf{E} = \mathbf{E}_0 \, \mathrm{e}^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \ ,$$

is incident on the interface at $z = 0$ between the two dielectrics. Consider the case where $\mathbf{E}$ is parallel to the plane of incidence. Then, starting with the boundary conditions on the fields as derived in class, derive expressions for the ratios E_0'/E_0 and E_0''/E_0, where E_0' is the amplitude of the electric field of the transmitted wave and E_0'' is the amplitude of electric field of the reflected wave.

2. A plane wave of frequency ω is incident normally (along $\hat{z}$) from vacuum on a semi-infinite slab of material with a *complex* index of refraction $n(\omega)$.

   (a) Show that the ratio of reflected power to incident power is

$$R = \left| \frac{1 - n(\omega)}{1 + n(\omega)} \right|^2 \ .$$

   (b) Show that the ratio of transmitted power to incident power is

$$T = \frac{4 \ \mathrm{Re} \ n(\omega)}{|1 + n(\omega)|^2} \ .$$

   Hint: R and T are the reflection and transmission coefficients, respectively, defined here as

$$R = \left| \frac{(\mathbf{S}'' \cdot \hat{z})}{(\mathbf{S} \cdot \hat{z})} \right| \qquad T = \left| \frac{(\mathbf{S}' \cdot \hat{z})}{(\mathbf{S} \cdot \hat{z})} \right| \ ,$$

   where $\mathbf{S}$, $\mathbf{S}'$, and $\mathbf{S}''$ are the time-averaged Poynting vectors for the incident, reflected, and refracted waves, respectively.