CLASSICAL ELECTRODYNAMICS I

Homework Set 1

January 27, 2017

1. Consider a uniformly charged spherical shell of negligible thickness, radius R, and total charge Q.

   (a) The charge density as a function of radial distance r can be written in various forms, *e.g.*: (i) $\rho(r) \propto \delta(r - R)$; (ii) $\rho(r) \propto \delta(r/R - 1)$; (iii) $\rho(r) \propto \delta(r^2 - R^2)$; (iv) $\rho(r) \propto \delta(r^2/R^2 - 1)$. Determine the correctly normalized proportionality constants in each case.

   (b) Use your results from part (a) to determine a correctly normally expression for the charge density of a uniformly charged *ellipsoidal* shell of negligible thickness, with semimajor axes a, b, and c, and total charge Q.

2. Using one-dimensional Dirac delta functions and, where appropriate, the step function defined as

$$\Theta(x) \equiv \int_{-\infty}^{x} \delta(x)\, dx = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$$

   express the following charge distributions as three-dimensional charge densities $\rho(\mathbf{r})$:

   (a) In cylindrical coordinates, a charge λ per unit length uniformly distributed over a cylindrical surface of radius b;

   (b) In cylindrical coordinates, a charge Q spread uniformly over a flat circular disk of negligible thickness and radius R;

   (c) In spherical coordinates, a charge Q spread uniformly over a flat circular disk of negligible thickness and radius R.

   Note that $d\Theta(x)/dx = \delta(x)$.