CLASSICAL ELECTRODYNAMICS I

Homework Set 7

March 13, 2015

1. Two point charges q and $-q$ are located on the z axis at $z = a$ and $z = -a$, respectively.

   (a) Find the electrostatic potential as an expansion in spherical harmonics and powers of r for both $r > a$ and $r < a$.

   (b) Next find the potential for $r \neq 0$ by keeping the product $p = 2aq$ constant and then taking the limit $a \to 0$. This limit corresponds to a point dipole along the z axis.

   (c) Give the charge density ρ of the two charges in part (a) in terms of Dirac delta functions expressed in spherical coordinates.

   (d) Suppose now that the two charges in part (a) are surrounded by a grounded conducting spherical shell of negligible thickness and radius b. The shell is concentric with the origin. Use the spherical Green function expansion to determine the electrostatic potential as an expansion in spherical harmonics and powers of r. As in part (b), also find the potential for $r \neq 0$ by keeping the product $p = 2aq$ constant and then taking the limit $a \to 0$.

   (e) Use your final result from part (d) to find the surface charge density $\sigma(\theta)$ induced on the sphere that surrounds the dipole.

2. Consider a point charge q between two infinite parallel conducting planes held at zero potential. The planes are located at $z = 0$ and $z = a$ in a cylindrical coordinate system, with the charge on the z axis at $z = z_0$, $0 < z_0 < a$.

   (a) Give the charge density ρ of the charge in terms of Dirac delta functions expressed in cylindrical coordinates.

   (b) Solve the Laplace equation in the charge-free space between the conducting planes and show that the potential has the form,

$$\Phi(\rho, z) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi z}{a}\right) K_0\left(\frac{n\pi\rho}{a}\right) \ .$$

(c) Evaluate the expansion coefficients A_n by solving the Poisson equation with methods analogous to those used in class to find the spherical Green function expansion.

(d) Calculate the induced surface charge densities $\sigma_L(\rho)$ and $\sigma_U(\rho)$ on the lower and upper planes, respectively.