CLASSICAL ELECTRODYNAMICS I

Homework Set 4

February 13, 2015

1. A hollow (non-grounded) spherical conductor of radius R is located with its center at the origin. The sphere contains a point charge q located on the positive z axis a distance a from the origin. Use the *method of Green functions* to find the potential $\Phi(z)$ at *all* points on the z axis.

2. Multiwire proportional chambers (MWPCs) or drift chambers are detectors commonly used in nuclear and particle physics to determine the trajectories of charged particles. In this problem, you are to derive an equation for the electric potential inside such detectors.

   Consider an infinite line of charge with charge per unit length λ. Assume that the line is along the z axis at $x = y = 0$. Use Gauss' law to determine the electric field at a distance r from the line of charge. Then show that the electric potential can be written as

   $$\Phi = \mathrm{Re}\left(-\frac{\lambda}{2\pi\epsilon_0}\ln z\right) + \text{constant} ,$$

   where here, $z = x + \mathrm{i}y = re^{\mathrm{i}\theta}$.

   Now consider an infinite array of parallel line charges located at $y = 0$, $x = 0, \pm s, \pm 2s, \cdots$. Each of the line charges has charge per unit length λ. Use the superposition principle and the result above to show that the potential can be put in the form,

   $$\Phi = \sum_{n=-\infty}^{\infty} \mathrm{Re}\left(-\frac{\lambda}{2\pi\epsilon_0}\ln z_n\right) + \text{constant} .$$

   Next show that this sum can be evaluated in closed form to give

   $$\Phi = -\frac{\lambda}{2\pi\epsilon_0}\,\mathrm{Re}\ln\left[\sin\left(\frac{\pi z}{s}\right)\right] .$$

   The following mathematical identity may be useful:

   $$\sin z = z\prod_{k=1}^{\infty}\left(1 - \frac{z^2}{k^2\pi^2}\right) .$$

Show that the potential is given by

$$\Phi(x,y) = -\frac{\lambda}{4\pi\epsilon_0}\, \ln\left(\sin^2\frac{\pi x}{s} + \sinh^2\frac{\pi y}{s}\right) \,.$$

What is the asymptotic form of the corresponding electric field for $y \gg s$?