CLASSICAL ELECTRODYNAMICS I

Homework Set 9

April 24, 2013

1. Starting with the differential expression

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi}\, d\mathbf{r}' \times \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}$$

for the magnetic induction at the point P with coordinate $\mathbf{r}$ produced by a current element $I\ d\mathbf{r}'$ at $\mathbf{r}'$, show explicitly that for a closed loop carrying a current I the magnetic induction at P is

$$\mathbf{B} = \frac{\mu_0 I}{4\pi}\, \nabla\Omega\ ,$$

where Ω is the solid angle subtended by the loop at the point P. This corresponds to a magnetic scalar potential,

$$\Phi_M = -\frac{\mu_0 I \Omega}{4\pi}\ .$$

The sign convention for the solid angle is that Ω is positive if the point P views the "inner" side of the surface spanning the loop; that is, if a unit normal $\mathbf{n}$ to the surface is defined by the direction of current flow via the righthand rule, Ω is positive if $\mathbf{n}$ points *away* from the point P, and negative otherwise. (*Hint:* You must convert the line integral into a surface integral.)

2. A cylindrical conductor of radius a has a hole of radius b bored parallel to, and centered a distance d from the cylinder axis ($d+b < a$). The current density $\mathbf{J}$ is uniform throughout the remaining metal of the cylinder and is parallel to the axis. Use Ampère's law,

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}\ ,$$

and the principle of linear superposition to find the magnetic induction $\mathbf{B}$ in the hole. Express $\mathbf{B}$ in terms of the vectors $\mathbf{J}$ and $\mathbf{d}$, where $\mathbf{d}$ is the vector from the center of the cylindrical conductor to the center of the hole.

3. Consider a rectangular loop carrying a counterclockwise current I that is centered at the origin. The two sides of the loop parallel to the x axis have length $2a$ and the two sides parallel to the y axis have length $2b$.

   (a) Determine the magnetic induction $\mathbf{B}$ for all points located on the z axis.

   (b) For points off the current loop, the magnetic induction can be written in terms of the magnetic scalar potential Φ_M given by $\mathbf{B} = -\nabla\Phi_M$, where $\nabla^2\Phi_M = 0$ (see problem 1). Discuss the symmetry properties of $\Phi_M(x, y, z)$.