CLASSICAL ELECTRODYNAMICS I

Homework Set 1

January 25, 2013

1. Using one-dimensional Dirac delta functions and, where appropriate, the step function defined as

$$\Theta(x) \equiv \int_{-\infty}^{x} \delta(x)\, dx = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$$

   Note that $d\Theta(x)/dx = \delta(x)$. Express the following charge distributions as three-dimensional charge densities $\rho(\mathbf{r})$:

   (a) In spherical coordinates, a charge Q uniformly distributed over a spherical shell of radius R;

   (b) In cylindrical coordinates, a charge λ per unit length uniformly distributed over a cylindrical surface of radius b;

   (c) In cylindrical coordinates, a charge Q spread uniformly over a flat circular disk of negligible thickness and radius R;

   (d) In spherical coordinates, a charge Q spread uniformly over a flat circular disk of negligible thickness and radius R.

2. In nuclear physics, pairs of nuclei in which the number of protons (Z) of the first nucleus equals the number of neutrons (N) of the second are called mirror nuclei. If we ignore the small mass difference in the proton and neutron, then the mass difference of the mirror pair equals the difference in their Coulomb energy (*i.e.*, their electrostatic potential energy).

   (a) Consider the nucleus to be a uniformly charged sphere of radius R and total charge Ze. Calculate an expression for W_0, the electrostatic potential energy of the nucleus.

   (b) Now consider the nucleus to consist of Z pointlike protons and N pointlike neutrons distributed uniformly in a sphere of radius R. Calculate an expression for W, the electrostatic potential energy in this case. In what limit does your expression for W approach the value of W_0?