

# CLASSICAL ELECTRODYNAMICS I

## Homework Set 7

November 8, 2017

1. Two point charges  $q$  and  $-q$  are located on the  $z$  axis at  $z = a$  and  $z = -a$ , respectively.
  - (a) Find the electrostatic potential as an expansion in spherical harmonics and powers of  $r$  for both  $r > a$  and  $r < a$ .
  - (b) Next find the potential for  $r \neq 0$  by keeping the product  $p = 2aq$  constant and then taking the limit  $a \rightarrow 0$ . This limit corresponds to a point dipole along the  $z$  axis.
  - (c) Give the charge density  $\rho$  of the two charges in part (a) in terms of Dirac delta functions expressed in spherical coordinates.
  - (d) Suppose now that the two charges in part (a) are surrounded by a grounded conducting spherical shell of negligible thickness and radius  $b$ . The shell is concentric with the origin. Use the spherical Green function expansion to determine the electrostatic potential as an expansion in spherical harmonics and powers of  $r$ . As in part (b), also find the potential for  $r \neq 0$  by keeping the product  $p = 2aq$  constant and then taking the limit  $a \rightarrow 0$ .
  - (e) Use your final result from part (d) to find the surface charge density  $\sigma(\theta)$  induced on the sphere that surrounds the dipole.