CLASSICAL ELECTRODYNAMICS I

Homework Set 6

November 3, 2017

1. A thin, flat, conducting, circular disk of radius R is located in the $z = 0$ plane with its center at the origin. The disk is maintained at a fixed potential V. In cylindrical coordinates (ρ, ϕ, z), the potential is $\Phi = \Phi(\rho, z)$ because of azimuthal symmetry. It may be shown that the charge density (on each side of the disk) is

$$\sigma(\rho) = \frac{2\epsilon_0}{\pi} \frac{V}{\sqrt{R^2 - \rho^2}}.$$

   (a) Using the equation above, determine the total charge Q of the disk.

   (b) Show that the potential on the z-axis is

$$\Phi(z) = A \tan^{-1}\left(\frac{R}{|z|}\right),$$

   and determine the constant A.

   (c) Expand your equation for $\Phi(z)$ as a power series in z. Then find $\Phi(r, \theta)$ the potential at any point for $r > R$ as an expansion in Legendre polynomials. Check your result in the limit $r \gg R$ to make sure that you determined the total charge correctly in part (a).