CLASSICAL ELECTRODYNAMICS I

Homework Set 5

October 27, 2017

1. A uniformly charged disk with surface charge density σ and radius R lies in the x-y plane with its center at the origin.

   (a) Solve the Laplace equation in cylindrical coordinates to determine the general form of the electrostatic potential for points not on the disk. Your solution should involve ordinary Bessel functions. The potential should be finite everywhere and satisfy symmetry conditions, but you do not need to determine the expansion coefficients explicitly.

   (b) Find a closed-form equation for the electrostatic potential $\Phi(z)$ on the z-axis.

   (c) Now determine your expansion coefficients in part (a) using the results from part (b) and the Laplace transform,

   $$\int_0^\infty \mathrm{e}^{-st}\left[\frac{a}{t}J_1(at)\right]dt = \sqrt{s^2+a^2} - s\ .$$

   Substitute your expansion coefficients into your result from part (a) to give the full expression for $\Phi(\rho, z)$ for points anywhere not on the disk.

2. Consider a point charge q between two infinite parallel conducting planes held at zero potential. The planes are located at $z = 0$ and $z = a$ in a cylindrical coordinate system, with the charge on the z axis at $z = b$, $0 < b < a$.

   (a) Give the charge density ρ of the charge in terms of Dirac delta functions expressed in cylindrical coordinates.

   (b) Solve the Laplace equation in the charge-free space between the conducting planes and show that the potential has the form,

   $$\Phi(\rho, z) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi z}{a}\right) K_0\left(\frac{n\pi\rho}{a}\right)\ .$$

   You do not need to determine the expansion coefficients explicitly.