CLASSICAL ELECTRODYNAMICS I

Homework Set 1

September 15, 2017

1. Suppose that the electrostatic potential produced by a point charge q was not Coulombic, but instead varied with distance in a manner determined by a specified scalar function $f(r)$:

$$\Phi = \frac{q}{4\pi\epsilon_0} f(r),$$

where r is the distance from the charge. The equation $\nabla \times \mathbf{E} = 0$ is assumed to be still valid so that we can still write $\mathbf{E} = -\nabla\Phi$.

   (a) Calculate the potential produced by an infinite flat sheet at $z = 0$ with uniform charge per unit area σ.

   (b) Use the result of part (a) to show that the associated electric field is

$$\mathbf{E}(z) = \hat{z}\frac{\sigma}{2\epsilon_0} z f(z).$$

2. Prove the *mean value theorem*: For charge-free space, the value of the electrostatic potential at any point is equal to the average of the potential over the surface of *any* sphere centered on that point.

3. Use one-dimensional Dirac delta functions to express the following charge distributions as three-dimensional charge densities $\rho(\mathbf{r})$.

   (a) In cylindrical coordinates, a total charge Q uniformly distributed over a thin ring of radius R. Express your result in terms of the charge per unit length λ and assume that the ring lies in the $z = 0$ plane with its center at the origin.

   (b) In cartesian coordinates, an infinite line of charge uniformly distributed with charge per unit length λ along the z-axis.

   (c) Carefully describe the charge distribution that corresponds to the three-dimensional density,

$$\rho(x, y, z) = \frac{\pi q}{a}\delta(x)\delta(y)\delta\left(\sin\frac{\pi z}{a}\right).$$

   Also rewrite the charge density in a more standard form for this charge distribution.

4. A thin metal sphere has radius R and total charge Q.

   (a) Calculate the capacitance of the sphere.

   (b) Determine the energy density of the electric field at a distance r from the sphere's center, where $r > R$.

   (c) Calculate the total energy of the electric field outside the sphere.