INTRODUCTION TO NUCLEAR AND PARTICLE PHYSICS

Homework Set 9

April 20, 2016

1. All known interactions conserve energy, linear momentum, total angular momentum ($\mathbf{J}$), electric charge (Q), baryon number (B), and the various lepton numbers (L_e, L_μ, L_τ). The strong and electromagnetic interactions both conserve parity (P) and quark flavor quantum numbers, such as strangeness (S) and the third component of isospin (I_3), but only the strong interaction conserves total isospin ($\mathbf{I}$). The weak interaction does not conserve parity or flavor quantum numbers; however, in weak interactions, decays that change strangeness or charm by more than one are strongly suppressed. Thus, an approximate selection rule for the weak interaction is $|\Delta S| \leq 1$.

   Find which of the following reactions are forbidden by one or more conservation laws. Give all violated laws in each case. If the reaction is allowed, indicate which interaction (strong, weak, or electromagnetic) is responsible. The symbol d represents the deuteron. For the purpose of this problem, assume that reactions which do not conserve the various lepton numbers are forbidden.

   (a) $\pi^- + d \to n + n$
   (b) $\Sigma^+ \to p + \gamma$
   (c) $\Lambda \to p + \pi^-$
   (d) $K^- \to \pi^- + e^- + \mu^+$
   (e) $\pi^- + p \to \Sigma^- + \Sigma^+$
   (f) $\Delta^+ \to p + \gamma$

2. The Gell-Mann–Nishijima formula for the electric charge of a hadron is $Q = I_3 + \frac{1}{2}Y$, where Y is a quantum number called the hypercharge. Prove this formula using the fact that hadrons are known to be bound states of quarks and antiquarks. Determine Y in terms of the baryon number B, and the various quark flavor quantum numbers (I_3, S, C, $\mathcal{B}$, T).

   Hint: Let $N_q = n_q - n_{\bar{q}}$, where n_q and $n_{\bar{q}}$ denote, respectively, the number of quarks and antiquarks in the hadron. Express Q, B, and the various flavor quantum numbers in terms of the appropriate N_q, and then solve for Q in terms of B and the various flavor quantum numbers.

3. Determine all allowed charge states of the final particles in the following *strong* reactions:

   (a) $\pi^- + p \to \pi + \pi + N$
   (b) $\pi^+ + p \to \pi + \pi + N$
   (c) $\pi^- + p \to \pi + K + \Lambda$
   (d) $\pi^+ + p \to \pi + K + \Lambda$
   (e) $\pi^- + p \to \pi + K + \Sigma$
   (f) $\pi^+ + p \to \pi + K + \Sigma$

4. (a) It is desired to produce the Ω^- hyperon (sss) by strong interactions involving $K^- + p$ interactions. Find the reaction that involves the minimum incident energy and that satisfies all conservation laws of the strong interaction.

   (b) Suppose now that it is desired by produce the Ω^- by strong interactions involving proton-proton collisions. Find the reaction that involves the minimum incident energy and that satisfies all conservation laws of the strong interaction.

   Possibly useful information about particle masses:

$M(\pi^\pm) = 139.57$ MeV	$M(p) = 938.27$ MeV	$M(e^\pm) = 0.51$ MeV
$M(\pi^0) = 134.97$ MeV	$M(n) = 939.57$ MeV	$M(\mu^\pm) = 105.66$ MeV
$M(K^\pm) = 493.65$ MeV	$M(\Lambda) = 1115.63$ MeV	$M(\Sigma^+) = 1189.37$ MeV
$M(K^0) = 497.67$ MeV		$M(\Sigma^0) = 1192.55$ MeV