INTRODUCTION TO NUCLEAR AND PARTICLE PHYSICS

Homework Set 7

March 30, 2016

1. Tabulate the possible m states of three quadrupole ($\ell = 2$) photons and their symmetrized contributions to show that the permitted states have $J^P = 0^+$, 2^+, 3^+, 4^+, and 6^+.

2. (a) Consider a deformed nucleus that has the shape of an ellipsoid of revolution:

$$\frac{x^2 + y^2}{a^2} + \frac{z^2}{b^2} = 1.$$

In spherical coordinates, we may describe this shape as a power series in spherical harmonics:

$$r = R_{\text{av}}(1 + \beta_2 Y_{20} + \beta_4 Y_{40} + \cdots),$$

where the $Y_{\ell 0}$ are spherical harmonics. If $a \approx b$, then we may approximate the shape by the equation,

$$r \approx R_{\text{av}}(1 + \beta Y_{20}).$$

Use this approximation to determine R_{av} and $\beta = \beta_2$ in terms of a and b by requiring that the approximation and the exact equation for the shape agree at $\theta = 0$ and $\theta = \pi/2$.

(b) If the charge density of the nucleus is uniform inside the ellipsoid and zero outside, then its intrinsic electric quadrupole moment (in units $e = 1$) is given by

$$Q_0 = \frac{2}{5} Z(b^2 - a^2),$$

where Z is the atomic number of the nucleus. Show that, when a and b are replaced in terms of R_{av} and β, you obtain

$$Q_0 = \frac{3}{\sqrt{5\pi}} R_{\text{av}}^2 Z\beta(1 + n\beta),$$

where n is a pure number that you should determine.

3. Consider the following rotational levels in ^{170}Hf: 0^+ (0.000 MeV), 2^+ (0.100 MeV), 4^+ (0.321 MeV), 6^+ (0.642 MeV), 8^+ (1.042 MeV), 10^+ (1.504 MeV), 12^+ (2.014 MeV), 14^+ (2.565 MeV), 16^+ (3.150 MeV), 18^+ (3.764 MeV), and 20^+ (4.417 MeV). If the moment of inertia I is a constant, then a plot of the energy E_J versus $J(J+1)$ should yield a straight line with the slope related to I. Make such a plot of E_J versus $J(J+1)$. Discuss what the plot implies about the actual moment of inertia for ^{170}Hf (be explicit).