INTRODUCTION TO NUCLEAR AND PARTICLE PHYSICS

Homework Set 5

March 2, 2016

1. (a) Solve the Schrödinger equation for the S-wave neutron-proton bound state (the deuteron) using the potential,

$$V(r) = \begin{cases} +\infty & \text{if } r < R_{\text{core}}, \\ -V_0 & \text{if } R_{\text{core}} \leq r \leq R, \\ 0 & \text{if } r > R. \end{cases}$$

Express your result for $u(r)$ in terms of k_1 and k_2, defined by $k_1 = \sqrt{2m(V_0 - B)/\hbar^2}$ and $k_2 = \sqrt{2mB/\hbar^2}$, respectively, where B is the deuteron binding energy.

(b) From continuity of $u(r)$ and $du(r)/dr$ at $r = R$, show that

$$k_1 \cot[k_1(R - R_{\text{core}})] = -k_2.$$

(c) Next find the solution of the Schrödinger equation for S-wave neutron-proton scattering. Express your result for $u(r)$ in terms of κ_1 and κ_2, defined by $\kappa_1 = \sqrt{2m(V_0 + E)/\hbar^2}$ and $\kappa_2 = \sqrt{2mE/\hbar^2}$, respectively, where $E > 0$.

(d) From continuity of $u(r)$ and $du(r)/dr$ at $r = R$ when $E > 0$, show that

$$\kappa_1 \cot[\kappa_1(R - R_{\text{core}})] = \kappa_2 \cot(\kappa_2 R + \delta_0).$$

(e) Define $R' \equiv R - R_{\text{core}}$, and show that the continuity equation for $E > 0$ becomes

$$\kappa_1 \cot \kappa_1 R' = \kappa_2 \cot(\kappa_2 R' + \delta_0'),$$

with $\delta_0' = \delta_0 + \kappa_2 R_{\text{core}}$; hence, for small κ_2, a_0' and r_0' satisfy an "effective range approximation"

$$\kappa_2 \cot \delta_0' \approx -\frac{1}{a_0'} + \frac{1}{2} r_0' \kappa_2^2.$$

Thus, we know that

$$a_0' = R' - \frac{\tan \kappa R'}{\kappa}$$

and

$$r_0' = R' - \frac{1}{3}\frac{R'^3}{a_0'^2} - \frac{1}{\kappa^2 a_0'},$$

with $\kappa = \sqrt{2mV_0/\hbar^2}$. Finally, show that the S-wave scattering length and effective range are given by

$$a_0 = a_0' + R_{\text{core}}$$

and

$$r_0 = r_0'\left(1 - \frac{R_{\text{core}}}{a_0}\right)^2 + \frac{2}{3}\frac{R_{\text{core}}^3}{a_0^2} + 2R_{\text{core}}\left(1 - \frac{R_{\text{core}}}{a_0}\right),$$

respectively.

(f) Given the approximate values, $2m = 939$ MeV, $\hbar c = 197.3$ MeV, $V_0 = 47.9$ MeV, $R = 1.92$ fm, and $R_{\text{core}} = 0.22$ fm, calculate values (to the nearest tenth of a fermi) for a_0 and r_0. Also calculate E_{cm} (the energy E at which δ_0 becomes zero). In the laboratory frame where the target nucleon is at rest, use your value of E_{cm} to determine E_{lab}, the corresponding kinetic energy of the projectile nucleon. (Use nonrelativistic kinematics and ignore the proton-neutron mass difference.)