INTRODUCTION TO NUCLEAR AND PARTICLE PHYSICS

Homework Set 4

February 24, 2016

1. Consider the deuteron as a proton and a neutron bound in a three-dimensional square-well potential of range R and depth V_0. The radial wave function is given by $u(r) = A \sin k_1 r$ for $r < R$ and by $u(r) = C \exp(-k_2 r)$ for $r > R$, where $k_1 = \sqrt{2m(V_0 - B)/\hbar^2}$ and $k_2 = \sqrt{2mB/\hbar^2}$. Here $R = 2.1$ fm, $B = 2.22$ MeV, and $2m = 939$ MeV.

   (a) Use the continuity of the wave function to prove that

   $$A^2 \sin^2 k_1 R = C^2 \exp(-2k_2 R)$$

   and use this result to show that

   $$A^2 = \frac{2k_2}{1 + k_2 R}.$$

   (b) Let $P(r_0)$ denote the fraction of the time that the neutron and proton spend separated by a distance greater than r_0, where $r_0 \geq R$. Show that we can write

   $$P(r_0) = P(R)\xi^n,$$

   where $\xi = \exp(-2k_2 R)$ and $n = (r_0/R) - 1$. Evaluate $P(r_0)$ numerically for $r_0 = R$, $2R$, $3R$, $4R$, and $5R$.

   (c) Show that the expectation value of the potential energy can be written as $\langle V \rangle = -V_0[1 - P(R)]$, and evaluate $\langle V \rangle$ numerically.

   (d) Use the fact that $E = -B = \langle T \rangle + \langle V \rangle$ to determine the expectation value of the kinetic energy, $\langle T \rangle$, numerically.

2. Consider the ^{4}He nucleus as a bound state of two deuterons.

   (a) Determine all allowed values of orbital angular momentum ℓ for the two deuterons.

   (b) Suppose that the deuteron-deuteron potential is approximated by

   $$V(r) = \alpha\hbar c \cdot \delta(r - R) + v(r),$$

where $v(r)$ is an attractive three-dimensional square-well potential with range R and depth V_0. The Dirac delta function term in the potential is included to represent the repulsive Coulomb interaction between the two deuterons, and α is the fine-structure constant ($\alpha^{-1} = 137.036$). We may choose $R = 1.7$ fm, which is the measured charge radius of ^{4}He.

Solve the Schrödinger equation for this potential (assume $\ell = 0$) to find the form of the radial wave function $u(r)$. What value of the energy E should be used in this model to describe the ground state of ^{4}He? (*Hint:* It is *not* equal to the binding energy of ^{4}He.) Finally, determine the well depth V_0 for this potential.