INTRODUCTION TO NUCLEAR AND PARTICLE PHYSICS

Homework Set 3

February 17, 2016

1. Suppose that the charge density of a nucleus with atomic number Z has the shape of a triaxial ellipsoid:

$$\rho(x, y, z) = \rho_0 \exp\left[-\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right)\right].$$

   (a) Determine the central density ρ_0 in terms of the total charge Ze and a, b, and c.

   (b) Calculate $\langle r^2 \rangle$, the mean square charge radius of the nucleus.

   (c) Calculate the electric quadrupole moment:

$$Q = \int (3z^2 - r^2)\rho \, d\tau.$$

   (d) For an ellipsoid of revolution, we have $a = b$ and we may write $c = \gamma a$, where γ is a dimensionless scale factor. Use the results of parts (b) and (c) to show that

$$Q = 2Ze\left(\frac{\gamma^2 - 1}{\gamma^2 + 2}\right)\langle r^2 \rangle.$$

   Calculate γ for ^{2}H ($r_{\text{ch}} = 2.1$ fm, $Q = +0.00288$ eb), ^{176}Lu ($Q = +8.0$ eb), and ^{209}Bi ($Q = -0.37$ eb). For the two heavy nuclei, calculate the charge radius r_{ch} from the approximation $r_{\text{ch}} = \sqrt{3/5} r_0 A^{1/3}$, where $r_0 = 1.2$ fm. Which of the three nuclei is the most nearly spherical? Which is the most deformed?

2. Consider a neutron as consisting of a proton plus a π^- meson in an orbital state with orbital angular momentum quantum number ℓ.

   (a) Determine the value of ℓ if total angular momentum and parity are conserved when the neutron "decays" into a proton and a pion. (*Hint:* The charged pion has spin-parity 0^- and its mass is 139.57 MeV.)

   (b) What is the orbital magnetic dipole moment of the configuration described in part (a)? Express your answer in nuclear magnetons.

(c) Is it possible to account for the observed neutron magnetic moment from such a model? Suppose that the neutron wave function consisted of two terms, one corresponding to a pointlike Dirac neutron and the other to a proton-plus-pion. What would be the relative sizes of the two pieces of the wave function? (Assume that the proton also behaves like an ideal Dirac particle.)

(d) Repeat the previous analysis for the proton magnetic moment; that is, consider the proton as part pure Dirac proton, plus part Dirac neutron with orbiting π^+ meson in the same orbital state ℓ as found in part (a). (*Hint:* You should find that the proton behaves like a pure Dirac proton about the same fraction of time that the neutron behaves like a pure Dirac neutron.)