INTRODUCTION TO NUCLEAR AND PARTICLE PHYSICS

Homework Set 2

February 10, 2016

1. (a) The measured charge radius for ^{12}C is 2.45 fm, that for ^{16}O is 2.72 fm, that for ^{28}Si is 3.08 fm, that for ^{32}S is 3.24 fm, and that for ^{40}Ca is 3.48 fm. For each of these five nuclei, find the mirror pairs with one or more and with one less nucleon than the reference even-even nucleus. For example, if ^{12}C is the reference nucleus, then find the mirror pairs with $A = 11$ and $A = 13$.

   (b) Now calculate the binding energy B (in MeV) and the binding energy per nucleon B/A (in MeV/nucleon) for each of the 25 nuclei in part (a). Recall that $m(^1\text{H}) = 938.78$ MeV, $m_n = 939.57$ MeV, and 1 u = 931.49 MeV. [Use $m(^{41}\text{Sc}) = 40.969449$ u and take the other atomic masses from the handout (Appendix C).]

   Compare B/A for the odd-A mirror pairs with the values for the reference even-even nuclei. Is there a systematic behavior? Also comment on any apparent systematic variation of B/A with A for these nuclei.

   (c) Show that if each of the mirror nuclei is approximated as a bound state containing A nucleons distributed uniformly within a sphere of radius R, then the Coulomb energy difference of the mirror pair is

$$\Delta E_C = \frac{3}{5}\frac{\alpha\hbar c}{R}(A-1),$$

   where α is the fine-structure constant. [*Hint:* Begin with the Coulomb term in the semi-empirical binding energy formula,

$$B_C(Z) = -\frac{3}{5}\frac{e^2}{4\pi\epsilon_0 R}Z(Z-1).]$$

   Now assume that the difference in binding energy for a mirror pair is equal to the Coulomb energy difference and use the equation for ΔE_C to calculate the nuclear radius R. Note that the charge radius of a uniformly charged sphere of radius R is given by $r_{\text{ch}} = \sqrt{3/5}R$. How do the *charge radii* determined for the mirror pairs compare

with the charge radii determined by electron scattering for the reference nuclei? [Compare with the values of charge radii from electron scattering given in Homework Set 1.]

2. Calculate neutron separation energies S_n (in MeV) for each of the following light odd-A nuclei: ^{9}Be, ^{13}C, ^{17}O, ^{21}Ne, ^{25}Mg, ^{29}Si, ^{33}S, ^{37}Ar,and ^{41}Ca. Note that for each of these nuclei, Z is even and $N = Z + 1$. [Use atomic masses from the handout (Appendix C).]

3. Calculate proton separation energies S_p (in MeV) for each of the nine nuclei listed in problem 2. Compare the values of S_p and S_n and briefly discuss any observed trends. [Use atomic masses from the handout (Appendix C).]