INTRODUCTION TO NUCLEAR AND PARTICLE PHYSICS

Homework Set 1

January 27, 2016

1. The charge form factors of light nuclei can be fitted well in the momentum transfer range, $q < 3$ fm^{-1}, by the form

$$F(q) = (1 - \alpha y + \beta y^2)\exp(-y),$$

where $y = (qb/2)^2$ and α, β, and b are non-negative constants for a given nucleus.

(a) Calculate an expression for the root-mean-square charge radius, $r_{\rm ch} = \langle r^2 \rangle^{\frac{1}{2}}$, in terms of α, β, and b.

(b) Derive an integral expression for the charge density $\rho(r)$ in terms of the (arbitrary) form factor $F(q)$. You will need to simplify a three-dimensional inverse Fourier transform.

(c) Calculate an expression for the charge density $\rho(r)$ for a nucleus having Z protons and described by the form factor given above.
Hint: The following integral should be helpful:

$$\int_0^\infty \exp(-a^2x^2)\cos bx\ dx = \frac{\sqrt{\pi}}{2a}\exp\left(-\frac{b^2}{4a^2}\right)$$

(d) The simple shell model (which will be discussed in class later this semester) allows us to predict values for α and β:

Nucleus	α	β
^{12}C	4/9	0
^{16}O	1/2	0
^{28}Si	6/7	4/35
^{32}S	11/12	11/60
^{40}Ca	1	1/5

The measured charge radius for ^{12}C is 2.45 fm, that for ^{16}O is 2.72 fm, that for ^{28}Si is 3.08 fm, that for ^{32}S is 3.24 fm, and that for ^{40}Ca is 3.48 fm. Use this information and the result of part (a) to evaluate the value of b (in fm) for each nucleus.

(e) The form factor given in this problem may have up to two diffraction minima (zeroes). For each of the five nuclei, use the information provided to determine numerical values (in fm^{-1}) for q_1 and q_2 where $F(q_1) = F(q_2) = 0$. Use your numerical values of q_1 for ^{12}C and ^{16}O to calculate (in degrees) the value of the electron scattering angle θ_e that corresponds to the first diffraction minimum for a beam of 420 MeV electrons.

(f) Sketch or plot the charge densities for ^{16}O and ^{40}Ca as determined from this problem. You should find a striking difference in their shapes near $r = 0$.