QUANTUM MECHANICS

Homework Set 7

March 31, 2014

1. A particle of mass m is placed in a *finite* spherical well:

$$V(r) = \begin{cases} -V_0, & \text{if } r \leq a, \\ 0, & \text{if } r > a. \end{cases}$$

   Find the ground state by solving the radial equation with $l = 0$. Show that there is no bound state if

$$V_0 < \frac{\pi^2 \hbar^2}{8ma^2} \ .$$

   Hint: Follow the example discussed in class for the finite square well in one dimension.

2. The raising and lowering operators change the value of m by one unit:

$$L_\pm f_l^m = A_l^m f_l^{m\pm 1} \ ,$$

   where A_l^m is some constant. Show that

$$A_l^m = \hbar \sqrt{l(l+1) - m(m \pm 1)} \ ,$$

   if the eigenstates f_l^m are normalized.

   Hint: First show that $L_\mp$ is the hermitian conjugate of $L_\pm$ (since L_x and L_y are operators for observables, you may assume that they are hermitian). Then use the following identity (derived in class):

$$\mathbf{L}^2 = L_\mp L_\pm + L_z^2 \pm \hbar L_z \ .$$

3. Construct matrix representations for the spin matrices $\mathbf{S}_x$, $\mathbf{S}_y$, and $\mathbf{S}_z$ for a spin-1 particle following the example in class for a spin-1/2 particle. That is, begin with matrix representations for the three spin states, $\alpha = |1, 1\rangle$, $\beta = |1, 0\rangle$, and $\gamma = |1, -1\rangle$, and then determine the action of $\mathbf{S}^2$, $\mathbf{S}_z$, and $\mathbf{S}_\pm$ on these states.