QUANTUM MECHANICS
Homework Set 5
March 6, 2014

1. Consider a particle of mass m in the one-dimensional finite square well potential with energy $E > 0$:

$$V(x) = \begin{cases} -V_0, & \text{if } -a \leq x \leq a, \\ 0, & \text{if } |x| > a. \end{cases}$$

Then the wave function has the form:

$$\psi(x) = \begin{cases} A\mathrm{e}^{\mathrm{i}kx} + B\mathrm{e}^{-\mathrm{i}kx}, & \text{if } x < -a, \\ C\sin(lx) + D\cos(lx), & \text{if } -a \leq x \leq a, \\ F\mathrm{e}^{\mathrm{i}kx}, & \text{if } x > a, \end{cases}$$

where we have assumed no incoming wave from $+\infty$.

(a) Use the boundary conditions on $\psi(x)$ and $d\psi(x)/dx$ to show that

$$C = F\mathrm{e}^{\mathrm{i}ka}\left[\sin(la) + \frac{\mathrm{i}k}{l}\cos(la)\right].$$

(b) Use the boundary conditions on $\psi(x)$ and $d\psi(x)/dx$ to show that

$$D = F\mathrm{e}^{\mathrm{i}ka}\left[\cos(la) - \frac{\mathrm{i}k}{l}\sin(la)\right].$$

(c) Use the equations in parts (a) and (b) to eliminate C and D from your boundary conditions to show that

$$F = \frac{A\mathrm{e}^{-2\mathrm{i}ka}}{\cos(2la) - \mathrm{i}\left(\frac{k^2+l^2}{2kl}\right)\sin(2la)}.$$

(d) Use the equations in parts (a) and (b) to eliminate C and D from your boundary conditions to show that

$$B = iF\left(\frac{l^2 - k^2}{2kl}\right)\sin(2la).$$

(e) Use the definition of the transmission coefficient, $T = |F|^2/|A|^2$, to show that

$$T^{-1} = 1 + \frac{V_0^2}{4E(E+V_0)} \sin^2\left(\frac{2a}{\hbar}\sqrt{2m(E+V_0)}\right).$$

(f) Use the definition of the reflection coefficient, $R = |B|^2/|A|^2$, to calculate R and show that $T + R = 1$.