# QUANTUM MECHANICS

## Homework Set 2

## February 6, 2014

1. In class, I showed that
$$\langle \mathbf{p} \rangle = m\frac{d\langle \mathbf{r} \rangle}{dt}.$$
Show that
$$\frac{d\langle \mathbf{p} \rangle}{dt} = \langle -\nabla V \rangle \ .$$
This is another example of *Ehrenfest's theorem*, which tells us that expectation values obey classical laws.

   *Hint:* The following theorem from vector calculus may be useful:
$$\int_V \nabla\psi \ d^3r = \int_S \psi \ d\mathbf{a} \ ,$$
where S is the closed surface bounding volume V and $d\mathbf{a}$ is an element of surface area outwardly normal to the surface.

2. Consider the time-independent one-dimensional Schrödinger equation:
$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi \ .$$
Show that if $V(-x) = V(x)$ then $\psi(x)$ can always be taken to be either an odd or even function of $x$.

   *Hint:* If $\psi(x)$ satisfies the Schrödinger equation for a given $E$, then so too does $\psi(-x)$, and hence also the even and odd combinations $\psi(x) \pm \psi(-x)$.

3. In classical mechanics, if you replace $V(x)$ by $V(x) + V_0$, where $V_0$ is constant in space and time, then this doesn't change anything (the force stays the same and we get the same solution $x(t)$ for the equation of motion). According to the correspondence principle, replacing $V(x)$ by $V(x)+V_0$ in quantum mechanics should have no effect on the expectation values. This will be true if the wave function only picks up a time-dependent phase factor, $\exp(\mathrm{i}at)$, where $a$ is a constant. Show that this is true and determine $a$ in terms of $V_0$.

4. Calculate (a) $\langle x \rangle$, (b) $\langle x^2 \rangle$, (c) $\Delta x$, (d) $\langle p \rangle$, (e) $\langle p^2 \rangle$, (f) $\Delta p$, and (g) $\Delta x \Delta p$ for the $n$th stationary state of the infinite square well. In part (g) check that the uncertainty principle is satisfied. What state comes closest to the uncertainty limit?