QUANTUM MECHANICS

Homework Set 1

January 23, 2014

1. A particle is represented at time $t = 0$ by the wave function

$$\psi(x, 0) = A\ \mathrm{e}^{-ax^2}\ \mathrm{e}^{\mathrm{i}bx}\ ,$$

where A, a, and b are real positive constants. If you are very clever, you should not need to look up any of the integrals needed for this problem.

(a) Determine the normalization constant A.

(b) What is the expectation value of x at time $t = 0$?

(c) What is the expectation value of x^2 at time $t = 0$?

(d) Find the uncertainty Δx in x at time $t = 0$.

(e) What is the expectation value of p at time $t = 0$?

(f) What is the expectation value of p^2 at time $t = 0$?

(g) Find the uncertainty Δp in p at time $t = 0$.

(h) Determine the product $\Delta x \Delta p$ at time $t = 0$ and verify that your results are consistent with the uncertainty principle, $\Delta x \Delta p \geq \hbar/2$. Comment on your result.

(i) The initial state wave function can be written as

$$\psi(x, 0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \phi(k)\ \mathrm{e}^{\mathrm{i}kx}\ dk\ .$$

Determine the k-space wave function $\phi(k)$ and comment on your result.