PHY 6/75101 October 27, 1997

Classical Mechanics — TEST I

Each question carries the same credit.

1. Use Lagrange's eq. to obtain the equation of motion for a particle constrained to move (a) vertically and (b) horizontally near the surface of the earth, assuming a dissipation function bv^2. In each case, assume an initial velocity v_0 and integrate to find $v(t)$.

2. (a) For a spherical pendulum (a rigid rod of negligible mass, with a mass m suspended from it), write down the Lagrangian in terms of two independent coordinates.
(b) Consider a double pendulum which consists of a mass m_2 at the end of a rigid weightless rod of length ℓ_2; this rod itself is suspended from a mass m_1 at the end of a rigid weightless rod of length ℓ_1. The system is constrained to move in a plane. Write down the Lagrangian in terms of independent coordinates.

3. A particle moves under an attractive central force

$$f(r) = -\frac{l^2}{mr^3}\left(1 + \frac{d^2}{r^2}\right)$$

where l is angular momentum and d is a constant. Find and sketch the equivalent one-dimensional potential V'. Show that a spiral path leads to this form of force. Discuss the possible motion.

4. Write a paragraph, keeping mathematical detail to a minimum, on each of the following:
(a) Holonomic and non-holonomic constraints.
(b) Uses of the Lagrange Multiplier method.
(c) Symmetry & conservation laws.