PHY 6/75101 October 1998

Classical Mechanics — Homework IV

Pass 1 for this homework is due by 10:00 a.m. Monday October 12.

A. The potential energy of a non-spherical satellite varies slightly, depending on its orientation relative to its orbit. Consider the simplest possible case — a satellite in the form of two identical spherical modules joined by an effectively massless frame. What is the stable orientation of this satellite?

B. A body moves under an attractive central force

$$f(r) = -\frac{\ell^2}{mr^3}\left(1 + \frac{d^2}{r^2}\right)$$

where ℓ is angular momentum and d is a constant. Find and roughly sketch the equivalent one-dimensional potential V'. Show that a spiral path of the form $r = r_0 + c\theta$ (where r_0 and c are constants) leads to this form of force. What can be concluded about the total energy of the body if it is known to follow a spiral path of the above form? Discuss the conditions under which the motion is or is not bounded.

C. A body moves in a spiral path of the (different!) form $r = c\theta^2$ under a central force specified by the potential

$$V(r) = Ar^n + Br^m\,,$$

where A and B are constants. Find the required values of the exponents n and m.

D. A planet of mass m orbits a star of mass M. A uniform distribution of dust with density ρ surrounds the star out to the planet's orbit.
(1) What is the central force?
(2) Consider a circular orbit of radius R with angular momentum ℓ. Find a polynomial equation connecting R and the other parameters given.
(3) Next consider small deviations from a circular orbit in the approximation of low dust density. Show that the natural frequency of oscillations is such that the perturbed orbit can be described by a precessing ellipse. Does the precessional angular frequency have the same or opposite sign as the orbital angular frequency?