PHY 6/75101 September 1998

Classical Mechanics — Homework III

The final version for this homework is due Wednesday Sep. 30.

A. The reflector mirror of a searchlight normally has a parabolic shape (or more precisely, is a paraboloid of revolution). Let z be the axis of revolution. The paraboloid can be described in cylindrical coordinates by $\rho^2 = az$, where a is a constant and ρ is the distance from the z axis to any point on the surface. Suppose that such a mirror points straight up, and a particle of mass m, total energy E, and angular momentum J, slides on it without friction. Use the Lagrange Multiplier method to find the magnitude of the constraint force as a function of ρ and the constants given above.

B. Now consider a particle sliding without friction on the surface of a different upward-facing bowl, this time having a spherical shape of radius R. Aside from R, we know only the particle's mass m.
(1) Determine the Lagrangian in terms of the usual angles θ and ϕ.
(2) Determine the generalized momenta p_θ and p_ϕ.
(3) Discuss cyclic coordinates and conserved quantities in the context of this example.
(4) If $\theta = \theta_0$ (a constant) at all times, find the velocity of the particle.

C. Let us go back to the Lagrangian considered in Homework II-B:

$$L = e^{bt}(\tfrac{1}{2}m\dot{q}^2 - \tfrac{1}{2}kq^2)$$

where b and k are each positive constants. Consider a transformation $s = q\exp(bt/2)$. Rework the problem in terms of the coordinate s. Explain any differences between the solutions in terms of q and s.