PHY 6/75101 December 18, 1997

Classical Mechanics — Final Exam

Each question carries the same credit.

1. Consider a system of particles in which the ith particle experiences a total force $\mathbf{F}_i + \mathbf{f}_i$: the first term represents conservative forces, and the second term is a frictional force linear in the velocity. Assume that the motion of the system is in a steady state, with energy being continually supplied to it as required to compensate for the frictional losses. Apply the virial theorem to this system.

2. Three identical point masses are located at cartesian coordinates $(x, y, z) = (3a, 0, 2a)$, $(0, 2a, 0)$, and $(2a, 0, 3a)$. Find the principal moments of inertia about the origin and a set of principal axes.

3. Find the eigenfrequencies of small vibrations for a symmetric diatomic molecule, and describe each normal mode. Normalize all eigenfunctions that arise in your solution.

4. For what conditions on the four constants α, β, γ and δ does

$$Q = q^\alpha p^\beta, \qquad P = q^\gamma p^\delta$$

represent a canonical transformation for a system with a single degree of freedom? Verify that $\alpha = 1$ either gives the identity transformation or a transformation that is not canonical.

5. Apply the Hamilton-Jacobi method to find the position as a function of time for a body which is released from rest at a height h above the surface of the earth. You may assume a constant acceleration g, and air resistance can be neglected.

6. Write a paragraph on any *TWO* of the following. Include mathematical details where necessary, but pay particular attention to the underlying concepts.
(a) The physical significance of *products of inertia* (off-diagonal elements in the inertia tensor of a body).
(b) The logistic map equation and Lyapunov exponents.
(c) Infinitesimal canonical transformations.