PHY 6/75101 November 28, 2007

Classical Mechanics — Homework #8

The due date for this homework is Friday December 7.

A. The Lenard-Jones Potential

$$V(r) = -\frac{A}{r^6} + \frac{B}{r^{12}}, \qquad A, B > 0$$

gives a description of the interaction between two isolated atoms which do not form a chemical bond, where r is the distance between the centers of the atoms. Consider a case where the reduced mass of the system is m and the total mass is M.
(i) Write down the Hamiltonian H in spherical polar coordinates.
(ii) Under what kind of scenario would the Hamiltonian depend explicitly on M and m, as opposed to m only?
(iii) If H is a minimum, what are the possible values of all the relevant phase space variables?
(iv) What is the frequency of small oscillations about the minimum energy?

B. Use the Poisson bracket formalism to show that the quantity

$$u(q, p, t) = \ln(p + i\,\omega m q) - i\,\omega t\,, \qquad \omega = \sqrt{k/m}\,,$$

is a constant of the motion in the case of a one-dimensional harmonic oscillator. Explain the physical meaning of this constant.

C. Two massless springs with force constants k_1 and k_2, and having the same equilibrium length ℓ, are attached end-to-end in a straight line. A particle of mass m is attached at the junction. The outer end of each spring is fixed rigidly at points separated by a distance a, and you may take $a \gg 2\ell$. Neither gravity nor friction need be considered.
(i) Find the Hamiltonian, H.
(ii) Is H conserved, and is the mechanical energy $T + V$ conserved?
(iii) Suppose we define a new coordinate

$$Q(q, t) = q - \frac{k_2 a}{k_1 + k_2} \sin \omega t\,.$$

Find the new Hamiltonian K in terms of the transformed coordinates. (Do not worry about simplifying the expression.)
(iv) Is K conserved? Is the mechanical energy $T + V$ conserved?