PHY 6/75101 October 26, 2007

Classical Mechanics — Homework #5

This homework is due Wednesday November 7.

A. We have already calculated the rate of precession for an elliptic orbit with small eccentricity if there is a non-inverse-square term in the central force. Show that Special Relativity predicts a precession of

$$\pi \left(\frac{GMm}{c\ell}\right)^2$$

radians per orbit (ℓ = angular momentum) for any elliptic orbit under a pure inverse-square force.

B. (i) Prove that for a rigid body, it is impossible for any one principal moment of inertia to exceed the sum of the other two.
(ii) Prove the Perpendicular Axis Theorem for principal moments. (First look up what it says, then prove it.)
(iii) For the cube problem worked in class, show that the principal moments $I_1 = M\ell^2/6$ and $I_2 = I_3 = 11M\ell^2/12$ follow from the determinant we wrote down.

C. Four equal masses m lie in a plane at coordinates

$$(x, y) = (d, 0),\ (-d, 0),\ (0, 2d),\ \text{and}\ (0, -2d)\,.$$

The masses are joined by massless rods to make a rigid body.
(i) Find the inertia tensor using the x, y, z Cartesian system.
(ii) Find the moment of inertia for rotation about an axis $\hat{\boldsymbol{\omega}}_\mathbf{0}$ which makes an equal angle with all three axes x, y, z.
(iii) If the body rotates about the $\hat{\boldsymbol{\omega}}_\mathbf{0}$ axis, find the angle between its angular momentum vector $\mathbf{L}$ and the $\hat{\boldsymbol{\omega}}_\mathbf{0}$ direction.

D. Find the principal moments and principal axes about the center of mass of a flat rigid body of uniform density in the shape of an isosceles triangle with one 90°-angle. Express your answer in terms of the mass of the body m and the length of the two equal sides ℓ.

E. If a rigid body is not subject to any net external torque, show that Euler's Equations of Motion lead to the conclusion that its rotational kinetic energy is conserved.

F. (i) Suppose that the radial pressure gradient at a distance of 15 km from the eye of a hurricane over New Orleans is 0.5 $\mathrm{N/m^3}$. What is the wind speed?
(ii) Compare and contrast the role of the Coriolis force in hurricanes, in tornadoes, and in the circulation of water draining from a sink or bathtub. Give plausible numerical examples to support your conclusions about the role of the Coriolis force in each situation.