PHY 6/75101 September 28, 2007

# Classical Mechanics — Homework #4

*The due date of this homework will be announced later.*

**A.** Three stars of unequal masses $m_1$, $m_2$ and $m_3$ can orbit about a common point and maintain a constant separation from each other if each one of them is the same distance $s$ from the other two. Show that a circular orbit with period

$$T = 2\pi s \sqrt{\frac{s}{G(m_1 + m_2 + m_3)}}$$

satisfies this condition.

**B.** (i) Suppose that we are dealing with a satellite in a low earth orbit, approximately circular. Suppose further that the residual atmosphere causes a slow but uniform rate of descent $dr/dt = -D$. Assuming that the drag force in this case is of the form $-bv^n\,\mathbf{v}$, show that only one numerical value of $n$ is possible, and find an expression for $b$ in terms of $D$ and the mass of the earth and the satellite.
(ii) How do you reconcile your finding in part (i) with conservation of angular momentum?

**C.** (i) Show that an orbit with a small but nonzero radial kinetic energy (i.e., it is nearly circular) has the form of a precessing ellipse if the net central force on the orbiting body can be represented by an inverse-square part and a small additional central force term $\mathcal{F}(r)$ which does not drop off like $1/r^2$.
(ii) Suppose $\mathcal{F}(r) = -kr^n$. For what conditions on $k$ and $n$ do the precessional angular frequency and the orbital angular frequency have the same sign?

**D.** The potential between an ion, with reduced mass $m$ and charge $q$, and a neutral atom at distances $r$ greater than the sum of their radii, can be written

$$V(r) = -\frac{q^2 P^2}{2r^4}\,,$$

where $P$ is a constant called the polarizability of the atom. Calculate the position of any maximum or minimum in the effective potential $V'$. (Give the position on both $V'$ and $r$ axes.) Then sketch $V'$ as a function of $r$, and discuss qualitatively the possible types of motion.