PHY 6/75101 September 14, 2007

Classical Mechanics — Homework #3

This homework is due by Friday September 21.

A. An "egg drop" is a game in which contestants use a selection of available materials to assemble a container inside which an egg is dropped from a height h onto a hard surface. The objective is to protect the egg from breaking. Suppose that $a_{\max}$ is the largest deceleration that an egg can withstand without breaking, and that the available cushioning material obeys Hooke's Law with an unknown spring constant up to a compressed thickness T that is a fairly small (but unknown) percentage of T_0, the original thickness of the cushioning layer. For values of T even smaller than this small percentage, the elastic limit of the material is exceeded and the effective spring "constant" increases very steeply. If the initial compression of the material causes a fraction f of the egg's incident kinetic energy to be dissipated as heat and the rest is initially converted to compressional potential energy, find an approximate expression for the needed minimum thickness $T_{\min}$ of the cushioning layer to protect an egg of unknown mass.

B. Masses m_1 and m_2 are joined by a massless spring with spring constant k and equilibrium length ℓ. The system moves on a frictionless horizontal rail (i.e., the motion is in one dimension only).
(i) Write down the Lagrangian for this system, and find the EOM and the general form of the motion $x_1(t) - x_2(t)$.
(ii) Consider the following initial conditions. Both masses begin at rest, then an impulse I is applied to m_1 at $t = 0$. The direction of the impulse is towards m_2. Find the expressions for $x_1(t) - x_2(t)$ and for $x_2(t)$ in terms of m_1, m_2, k, ℓ and I.
(iii) How far does m_2 move before coming momentarily to rest for the first time?

C. A wooden log of density ρ, radius R and length L rolls down a hill without slipping. The slope makes a constant angle ϕ with the horizontal.
(i) Choose a Cartesian system and write down the constraints for rolling, and for the log being in contact with the hill. Are either or both of these constraints holonomic?
(ii) Identify a set of generalized coordinates where the latter constraint disappears from the problem, but not the former, and write down the resulting Lagrangian.
(iii) Use the Lagrange Multiplier method to calculate this force of constraint. What is the physical origin of this force? (For example, the physical origin of the force of constraint that keeps the mass at a constant distance from the point of suspension in a pendulum is the force of tension in the rod or string.)

D. A body of mass m moves without friction on the surface of a planet of radius R.
(i) Write down the Lagrangian, the generalized momenta, and the Hamiltonian for this body.
(ii) Discuss cyclic coordinates and conserved quantities in the context of this example.
(iii) Show that the path of the body must follow a great circle.