PHY 6/75101 August 31, 2007

Classical Mechanics — Homework #2

This homework is due by class time on Monday September 10.

A. Consider a simple pendulum with mass m and length ℓ, whose point of support is in accelerated motion. The coordinates of the point of support are $(x_s,\, y_s)$.
(i) Use the Lagrangian to find the equation of motion (EOM) for θ, the angle of the bob of the pendulum. Do not assume small oscillations. You are not asked to solve the EOM.
(ii) For the special case of small oscillations where $y_s = 0$ and $x_s = x_0 \sin \omega_s t$, show that your result from part (i) reduces to the EOM for a driven harmonic oscillator.

B. Two unequal masses m and m' are connected by a string of length ℓ, threaded through a hole in a table. The mass m' is situated below the table, and is constrained by a suitably-sized tube to move in the vertical direction only.
(i) Neglecting friction, use the Lagrangian to find the equation of motion (EOM) for the mass m'. It is not necessary to solve the equation.
(ii) Symmetry arguments tell us that the motion of m' should not depend on the coordinate θ (polar angle) of the mass m on the plane of the table. Show that this conclusion is implied by your EOM.
(iii) Show that your EOM is compatible with a special form of motion where the mass m moves with constant speed v_R along a circular path of radius R, while m' remains motionless.
(iv) Is your EOM also compatible with a special solution where both masses are motionless, with m right over the hole (it is not small enough to fit through it), and m' hanging a distance ℓ below m?

C. (i) Two identical simple pendulums, each with mass m and length ℓ, have their masses joined by a massless spring of constant k. The spring is at its equilibrium length when both pendulums are vertical. Write down the Lagrangian for this system and use Lagrange's equations to find the two EOM in the limit of small oscillations. It is not necessary to solve the equations.
(ii) Repeat the above for the case of an infinite row of coupled simple pendulums.