Physics 23102 **Fall 2000**

Ampère's Law

Any electric current i sets up a magnetic field $\mathbf{B}$ and Ampère's law can be regarded as a general description of the relationship between $\mathbf{B}$ and i, very much analogous to Gauss' law which we have used to relate electric field to electric charge.

$$\oint_P \mathbf{B} \cdot d\mathbf{s} = \mu_0 \, i_{enc}$$

where the line integral on the left is evaluated over ANY closed path P, and i_{enc} is the net current enclosed by the path P. Note that P is a purely mathematical construction, and does not need to coincide with any real conductor, or any real field, etc. The constant $\mu_0 = 4\pi \times 10^{-7}$ T $\cdot$ m/A is called the *permeability of free space*. Currents **not enclosed** by P contribute to $\mathbf{B}$ but their contribution to $\oint_P \mathbf{B} \cdot d\mathbf{s}$ cancels to zero (see example worked in class).

If our problem does not have a sufficient degree of symmetry, Ampère's law is still true, but it doesn't help us to find $\mathbf{B}$ starting from a known configuration of current. If symmetry *is* present, Ampère's law gives us a very easy way to calculate B. After you have worked several examples, you will recognize the general approach — symmetry allows us to bring $\mathbf{B}$ outside the integral so that it becomes $B \int_{P_\parallel} ds$ and then $\int_{P_\parallel} ds$ is simply the length of that part of P where $B \neq 0$ and the vectors $\mathbf{B}$ and $\mathbf{s}$ are parallel or antiparallel. (Again, notice the similarity to what we do with Gauss' law.)

When using Ampère's law to find B, the desired path P is normally either a circle or a rectangle. In the case of circular symmetry, the integral reduces to $B\ 2\pi r$. If the magnetic field is uniform over a certain region of space, we normally need a rectangular path, in which case we write

$$\oint_P \mathbf{B} \cdot d\mathbf{s} = \int_a^b \mathbf{B} \cdot d\mathbf{s} + \int_b^c \mathbf{B} \cdot d\mathbf{s} + \int_c^d \mathbf{B} \cdot d\mathbf{s} + \int_d^a \mathbf{B} \cdot d\mathbf{s}\,,$$

where a, b, c and d represent the four corners of the rectangular path P. Typically, three of the four sides contribute zero.

EXERCISE:

A toroidal coil has inner radius r_1, outer radius r_2, has a total of N turns, and carries a steady current i_0.

(a) Draw two concentric circles to represent the coil former, then sketch the first and last loops of the coil, and choose a direction for i_0.

(b) In the sketch, show one representative line of $\mathbf{B}$.

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(c) In order to evaluate the magnitude of $\mathbf{B}$ using Ampère's law, what is the simplest and most convenient path of integration, P? Draw P on a duplicate of your sketch of the coil former.

(d) Explain your reasoning for choosing this path P.

(e) The integral $\oint_P \mathbf{B} \cdot d\mathbf{s}$ can be thought of as the limit of the sum $\sum_{i=1}^{M} \mathbf{B}_i \cdot \delta\mathbf{s}_i$ as M becomes very large and each $\delta\mathbf{s}_i$ becomes very small. For the case $M = 8$, sketch the eight $\delta\mathbf{s}$ vectors as arrows on a new duplicate of your coil former.

(f) Taking the path P you drew in part (c), evaluate the integral $\oint_P \mathbf{B} \cdot d\mathbf{s}$.

(g) In terms of symbols given at the beginning of this exercise, what is i_{enc} for your chosen path P?

(h) What is i_{enc} for a circular path of radius $r < r_1$?

(i) What is i_{enc} for a circular path of radius $r > r_2$?

(j) Using your results from (f) and (g), write the expression for $B(r)$.

(k) On a new duplicate of your coil former, sketch a small rectangular path *abcd*, such as we used when calculating B inside a solenoid.

(l) Show that the path *abcd* leads to the same expression for $B(r)$ as you obtained in part (j).