Spring 2010

Thermal and Statistical Physics

1. (25 pts) N non-interacting spin ½ atoms are placed in an external magnetic field H; each atom has a magnetic susceptibility μ. Calculate, using the microcanonical ensemble, the average magnetic moment per atom as a function of temperature. You may find Stirling's approximation useful: $\log(N!) \approx N(\log(N)-1)$

2. (20 pts) Without making detailed calculations, estimate the following (techniques you may find useful include dimensional analysis, equivalence of length and/or energy scales :
   a. The number density of a gas at which quantum mechanical effects become important
   b. The heat capacity of an ideal gas
   c. The temperature at which the rotational degree of freedom "freezes out" for a diatomic molecule

3. (20 pts) Calculate the density of states for a hypothetical spinless particle with dispersion relation: $\epsilon = \hbar|k|^{\gamma}$.

4. (20 pts) The free electron model of metals assumes the electrons can be treated as an ideal gas occupying volume V at temperature T. The average number of electrons with energy ε is given by the Fermi-Dirac distribution:

$$n(\varepsilon) = \frac{1}{\exp[\beta(\varepsilon-\mu)]+1}$$

   a. Sketch $n(\varepsilon)$ for T=0 and $T > 0$.
   b. If the density of states is $A\varepsilon^{1/2}$, calculate the Fermi energy. Your result should be expressed in terms of N and A.
   c. What is the physical significance of the Fermi energy?

5. (15 pts) A quantity of ideal gas in a thermally isolated box is placed in a rocket ship on the surface of a massive planet. The rocket then takes off, and travels a great distance from the planet. Does the temperature of the gas increase, decrease or remain the same? You can construct your answer for this question without using any equations.