1. (25 pts) $N$ non-interacting spin $\frac{1}{2}$ atoms are placed in an external magnetic field $H$; each atom has a magnetic susceptibility $\mu$. Calculate, using the microcanonical ensemble, the average magnetic moment per atom as a function of temperature. You may find Stirling’s approximation useful: $\log(N!) \approx N\left(\log(N) - 1\right)$.

2. (20 pts) Without making detailed calculations, estimate the following (techniques you may find useful include dimensional analysis, equivalence of length and/or energy scales):
   a. The number density of a gas at which quantum mechanical effects become important
   b. The heat capacity of an ideal gas
   c. The temperature at which the rotational degree of freedom “freezes out” for a diatomic molecule

3. (20 pts) Calculate the density of states for a hypothetical spinless particle with dispersion relation: $\epsilon = \hbar |k|^\gamma$.

4. (20 pts) The free electron model of metals assumes the electrons can be treated as an ideal gas occupying volume $V$ at temperature $T$. The average number of electrons with energy $\epsilon$ is given by the Fermi-Dirac distribution:

   $$n(\epsilon) = \frac{1}{\exp[\beta(\epsilon - \mu)] + 1}$$

   a. Sketch $n(\epsilon)$ for $T=0$ and $T>0$.
   b. If the density of states is $A\epsilon^{3/2}$, calculate the Fermi energy. Your result should be expressed in terms of $N$ and $A$.
   c. What is the physical significance of the Fermi energy?

5. (15 pts) A quantity of ideal gas in a thermally isolated box is placed in a rocket ship on the surface of a massive planet. The rocket then takes off, and travels a great distance from the planet. Does the temperature of the gas increase, decrease or remain the same? You can construct your answer for this question without using any equations.