Candidacy Exam Fall 2006

Statistical Mechanics

[30 pts] 1. Consider a system which has three particles that can occupy one of two states with energies 0 and ϵ. Write down the canonical partition functions corresponding to:

(a) distinguishable particles (with classical statistics)
(b) spin 0 Bosons.
(c) spin 1/2 Fermions.

Compare the mean energies of each system (a–c) in the limits:

(d) $T \to \infty$
(e) $T \to 0$

[35 pts] 2. A (hypothetical) substance can be in two phases with free energies

$$F_1 = \frac{1}{8}\frac{a}{T}\frac{N^2}{V} \quad \text{and} \quad F_2 = \frac{1}{9}\frac{b}{T}\frac{N^3}{V^2}. \tag{1}$$

where a and b are constants. Find the pressure, P, and densities $\rho_1 = N_1/V_1$ and $\rho_2 = N_2/V_2$, when the two phases coexist in equilibrium.

[35 pts] 3. Consider a gas of non-interacting bosons in a volume V above the Bose condensation temperature, T_0. The dispersion relation for the single particle energy states is given by $\epsilon(p) = Ap^\sigma$ where p is the magnitude of the momentum and σ is some constant.

(a) Show that the pressure, P, and energy, E, are related through

$$P = \alpha\frac{E}{V} \tag{2}$$

and find the proportionality constant α.

(b) Is this relationship valid below the Bose condensate temperature ($T < T_0$)? (Note: you can answer this part without answering part (a).)