Candidacy Exam Fall 2003

Statistical Mechanics

25 pts. 1. For blackbody radiation at temperature T in a box of volume V, it is found that $U = aV T^4$ and $P = a T^4/3$ where a is a constant. Obtain an expression for the entropy in terms of U and V. (The chemical potential is 0.)

25 pts. 2. A system consists of N atoms with one atom on each of N lattice sites. Atoms cannot exchange places. Each atom has three energy levels: E, 2E, and 3E. Write down the canonical partition function and calculate the entropy, S and energy, U. Sketch S(T) and U(T).

25 pts. 3. Consider a three dimensional ideal gas of N spin ½ fermions in volume V.

a) Find the chemical potential at T = 0.

b) Calculate the temperature at which the chemical potential is zero.

c) Graph the chemical potential as a function of temperature.

$$\left[\int_0^\infty d\,x\, \frac{x^{\frac{1}{2}}}{(e^x + 1)} = 0.6780 \text{ may be useful.} \right]$$

25 pts. 4. If fluctuations are neglected, the transition from the paramagnetic state to the ferromagnetic state as temperature is reduced can be described by a Helmholtz free energy of the form $F = F_p - k M^4$ where F_p is the free energy of the paramagnetic state, k is a positive constant, and M is the magnetization. The transition occurs at $T = T_c$, $M = 0$ for $T > T_c$ and $M = b (T_c - T)^{1/2}$ for $T < T_c$, where b is a constant.

a) Calculate and sketch the specific heat c(T),

b) Sketch the specific heat when the effect of fluctuations is included. (You do <u>not</u> need to do any calculation.)