CANDIDACY EXAM – SPRING 2013

QUANTUM MECHANICS

Total: 100 Points

*** Attempt all the problems. There are several parts to each problem with unequal weights, shown in parentheses.

1. [35 points]

Imagine that a particle is subjected to a potential which allows it to move freely on the x-axis except that it *may not* be at $x < -L/2$ or $x > +L/2$.

[10 p] (a) Verify if the operators $\hat{P} = -i\hbar\frac{d}{dx}$ and $\hat{P}^2$ are *Hermitian* by considering arbitrary wave functions, ψ and ϕ ($\psi \neq \phi$), appropriate for the problem.

[16 p] (b) Obtain the the *eigenfunctions* and *eigenvalues* of $\hat{P}^2$ in this space ? Comment on the *parity* of these eigenfunctions. Does $\hat{P}$ have *eigenfunctions* in this space ? Justify your answer.

[9 p] (c) What are the lowest three *eigenvalues* of $\hat{P}^2$ above ? Show the three appropriately spaced eigenvalues, together with sketches of the corresponding three *eigenstates*, on a plot of eigenvalue spectrum.

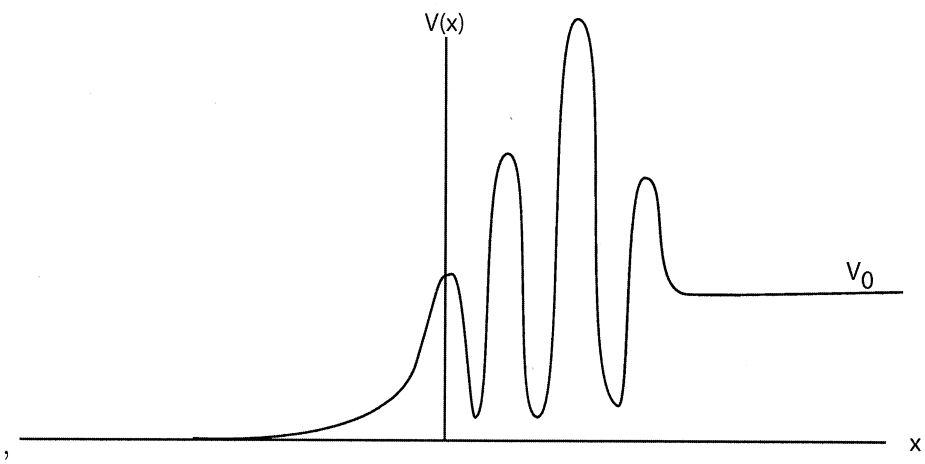
<u>Note:</u> You may find it useful to recall the following formula for integration by parts:

$$\int udv = uv - \int vdu$$

2. [30 points]

A particle of mass m experiences the following potential in 1-dimension (see sketch below):

$$V(x) = \begin{cases} \to 0, & \text{for } x \to -\infty \\ \text{"something ghastly" (as in figure)}, & \text{for other } x \text{ values} \\ \to V_o > 0, & \text{as } x \to +\infty \end{cases}$$



Consider the particle to be incident from the left ($x \to -\infty$) with definite momentum p, and energy $E > V_o$.

[10] (a) Find the form of the wavefunction of the particle for $x \to -\infty$ and $x \to +\infty$.

[8] (b) Find the probability *current* density for $x \to -\infty$ and $x \to +\infty$ using the wavefunctions found in part (a).

[12] (c) Using the continuity equation for probability and (b), derive a relation between the reflection and transmission coefficients.

3. [35 points]

Consider a spin 1/2 electron at rest in a *uniform* magentic field, $\vec{B} = B_z \hat{k}$. Then the Hamiltonian is given by:

$$H_o = \frac{e}{m} B_z \, S_z$$

You may recall that in this case the *eigenvalues* are given by:

$$E_{a,b} = \mp \frac{e}{m} B_z \frac{\hbar}{2}$$

and the corresponding *eigenvectors* by:

$$\chi_a = \begin{pmatrix} 1 \\ 0 \end{pmatrix} ; \quad \chi_b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Now turn on a *perturbation* H_1 in addition to H_o given by:

$$H_1 = \frac{e}{m} B_x \, S_x$$

<u>Find</u> the *variational upper bound* on the ground state energy of the *perturbed* system using the following *trial wavefunction*:

$$\chi = (\cos\phi) \, \chi_a \; + \; (\sin\phi) \, \chi_b$$

where ϕ is the adjustable parameter. First, <u>verify</u> if χ is normalized.

<u>Note</u>: You may want to evaluate the needed matrix elements in the Dirac "bra-ket" notation. You may find the following relations useful:

(a) Pauli spin matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(b) Trigonometric relations:

$$\sin 2\phi = 2 \sin\phi \cos\phi$$

$$1 + \cos 2\phi = 2 \, \cos^2\phi \, ; \qquad 1 - \cos 2\phi = 2 \, \sin^2\phi$$