Spring 2008

QUANTUM MECHANICS

1)(40pts) The following questions require only short answers, not lengthy calculations, but be clear in your answers.

a) (5pts) The energy spectrum of a one-dimensional harmonic oscillator is $E_n = (n + 1/2)\hbar\omega$ where $n = 0, 1, 2, 3....$ Why can't E_n be zero, even in the case of $n = 0$?

b) (5pts) Write down a similar expression for the energy spectrum of a three dimensional isotropic harmonic oscillator. What is the degree of degeneracy of the first four states?

c)(5pts) Let B and C be two anti-commuting operators, i.e. $\{B, C\}_+ \equiv BC + CB = 0$. Let Ψ be an eigenstate of both B and C. What can be said about the corresponding eigenvalues?

d) (5pts) For B = baryon number and C = charge conjugation, the relations $\{B, C\}_+ = 0$ and $C^2 = 1$ hold. What does your result in part (c) imply in this case?

e)(5pts) Two identical **fermions** inside a one dimensional infinite square well are interacting with one another via the hypothetical potential $V(x_1, x_2) = V_0\delta(x_1 - x_2)$, where V_0 is a positive constant. If we ignore spin effects what is the expected magnitude of the ground state perturbation and why? (No calculations are allowed).

f) (5pts) Mesons are made up of a quark-antiquark pairs and Baryons of three quarks. Quarks are fermions with spin= 1/2. What are the possible spin states for mesons and baryons?

g) (10pts) Find the eigenvalues and (normalized) eigenvectors of the following Hamiltonian:

H= $\begin{pmatrix} k & g \\ g & k \end{pmatrix}$, where k and g are real constants.

2)(30pts) Three non-interacting particles of mass m are confined to move on a circle of radius R. Their mutual distances are fixed and equal so that they form an equilateral triangle. The three particles obey Bose statistics and have no spin.

a) (10pts) Find an expression for the Hamiltonian of the system (Hint: this can be greatly simplified by a proper choice of the coordinate system).

b) (20pts) Find and discuss the rotational energy of the system.

3) (30pts) Consider a one-dimensional model of conduction electrons inside a slab of metal occupying $x < 0$. The interaction between the crystal and the conduction electrons is approximated by a step potential: $V(x) = -V_0$ where $(V_0 > 0)$ if $x < 0$ and $V(x) = 0$ if $x > 0$.

a) (10pts) Setup the problem and find the electron wavefunctions for the two regions.

b) (10pts) Calculate the probability of reflection and transmission of a conducting electron approaching the surface of the metal with total energy $E > 0$.

c) (10pts) Discuss your results in b) and calculate the reflection probability for the commonest metals where $V_0 \approx 10$ eV and electron energy is $E = 0.1$ eV. Compare your result with the classically anticipated one.