Quantum and Atomic Physics

1. (25 points) A particle of mass m is in a one-dimensional box with walls at $x = 0$ and $x = L$. (The energy eigenvalues and eigenfunctions are

$$E_n = \frac{\hbar^2 \pi^2}{2mL^2} n^2 \quad , \quad \psi_n(x) = \sqrt{\tfrac{2}{L}} \sin \tfrac{n\pi x}{L} \quad , \quad n = 1, 2, 3, \ldots\ .)$$

(a) Use the uncertainty principle to estimate the ground state energy of the system.

For parts (b), (c), and (d), the particle is in the initial state

$$\psi(x) = Ax(L - x) \ .$$

(b) Determine the value of the constant A.

(c) How does ψ evolve in time? (You don't have to carry out the integrals.)

(d) The energy is measured at $t > 0$. What is the probability of finding E_3? (You don't have to carry out the integral.)

2. (25 points) An electron in a periodic potential is described by a "Bloch wave function,"

$$\psi(x) = e^{ikx} u(x) \ ,$$

where $u(x)$ is a periodic function. ($\psi(x)$ is properly normalized.)

(a) What physical picture can you attach to this wave function?

(b) Calculate the expectation value of the momentum of the electron in this state.

(c) What is the expectation value of the momentum when $u(x)$ is real?

3. (30 points) An electron is in a time-independent, homogeneous magnetic field $\vec{B}$. Let the spin wave function at time $t = 0$ be

$$\psi(t=0) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

in a reference frame whose z axis is aligned with the magnetic field, $\hat{z} \| \vec{B}$.

(a) Solve the equation of motion for $\psi(t)$, i.e. find $\psi(t > 0)$.

(The magnetic moment of the electron is given by $\vec{\mu} = \frac{e\hbar}{2mc}\vec{\sigma}$.)

(b) Calculate the expectation values of the three components of the electron spin operator as functions of time, and interpret the result physically.

Note: recall the Pauli matrices,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1}$$

4. (20 points) Find the energy eigevalues of a particle of mass m moving in the potential

$$V(x) = \begin{cases} \infty & \text{if } x < 0 \\ \frac{1}{2}m\omega^2 x^2 & \text{if } x \geq 0 \end{cases}$$

Make a sketch of the two lowest energy eigenfunctions. (You may use what you know about the linear harmonic oscillator without derivation.)