Quantum Mechanics

Potentially useful information:

$\hbar c = 0.19733$ GeV • fm
1 GeV = 10^6 eV; 1 fm = 10^{-15} m

$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$

$\int \cos^3 x dx = \sin x - \frac{1}{3} \sin^3 x$

$\int \sin^2 x \cos x \, dx = \frac{1}{3} \sin^3 x$

Problems:

(15 pts.) 1. In an electron accelerator, an electron beam of 10GeV energy is used to study the structure of a target nucleus (at rest). What is the deBroglie wave length of the electrons, and why is it useful to work with beam energies of this order of magnitude?

2. Suppose that the energy of a particle in a one-dimensional box was measured, and the ground-state energy $E_1^{(a)} = \frac{\hbar^2\pi^2}{2ma^2}$ was obtained. We then know that the particle is in the ground state,

$$< x \,|\, E_1^{(a)} = \frac{\hbar^2\pi^2}{2ma^2} > \; = \; \psi_1(x) = \sqrt{\frac{2}{a}} \cos \frac{\pi x}{a} \quad \text{if } |x| < \frac{a}{2} \qquad .$$

(The walls of the box are at x = ± a/2.) Now assume that the walls are suddenly moved to x = ±a (the particle has no time to adjust).

(25 pts.) a.) What is the probability that a subsequent measurement of the energy will find the particle in the ground state of the enlarged box?

(10 pts.) b) What is the probability that a subsequent measurement of the energy will find the particle in the first excited state of the enlarged box with energy $E_2^{(2a)} = \frac{\hbar^2 \pi^2}{2m a^2}$

(10 pts.) c) How does the system (in the enlarged box) evolve with time? (Express your answer in terms of the energy eigenvalues and eigenstates of the enlarged box.)

3. The y and z components of the spin operator of a spin-1 particle can be represented by the following 3x3 matrices:

$$\hat{S}_y \Rightarrow \frac{\hbar}{\sqrt{2}}\begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix} ; \hat{S}_z \Rightarrow \hbar \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(10 pts.) a.) What matrix corresponds to $\hat{S}_x$ of the spin-1 particle in this representation?

(5 pts.) b.) Calculate the total spin squared ($\hat{\mathbf{S}}^2$) operator!

(5 pts.) c.) What are the common eigenvectors of $\hat{S}_z$ and $\hat{\mathbf{S}}^2$? What other subsets of these 4 operators have common eigenstates?

(10 pts.) d.) Calculate the action of the $\hat{S}_x + i\,\hat{S}_y$ operator on the common eigenvectors of $\hat{S}_z$ and $\hat{\mathbf{S}}^2$.

(10 pts.) e.) Apply the general Heisenberg uncertainty principle to the $(\hat{S}_x, \hat{S}_y)$ pair of operators and interpret your result.